1. Introduction to wall-bounded turbulent flows

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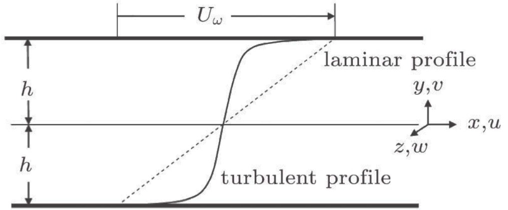
Les Houches School of Physics

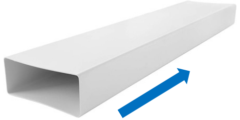
ÉCOLE DE PHYSIQUE DES HOUCHES

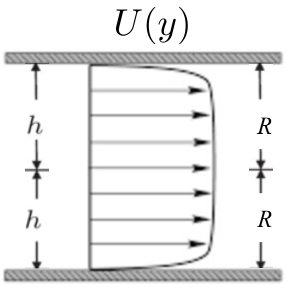
Canonical wall-bounded flows



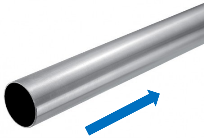
Couette flow

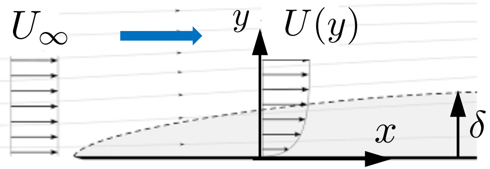
channel





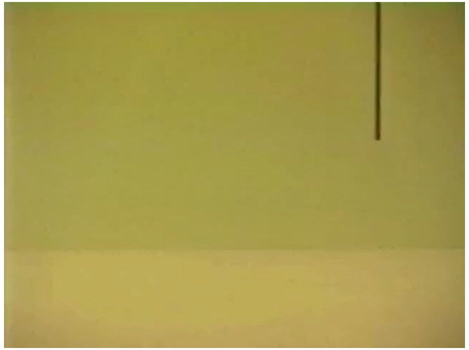
pipe





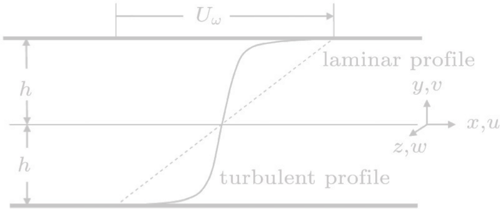
boundary layer

- Plane Couette flow
- Fully-developed channel flow of high aspect ratio
- Fully-developed pipe flow
- Boundary layer, flat plate, zero pressure gradient
- Ekman layers, Taylor-Couette flows, Rayleigh-Bénard convection, …
- All flows turbulent (high Reynolds number) and free of history effects

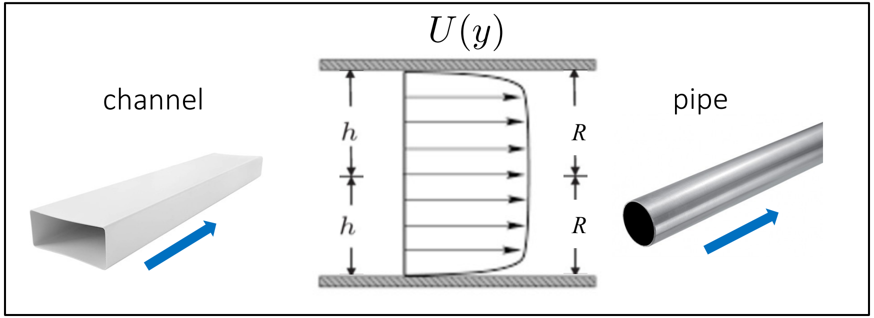


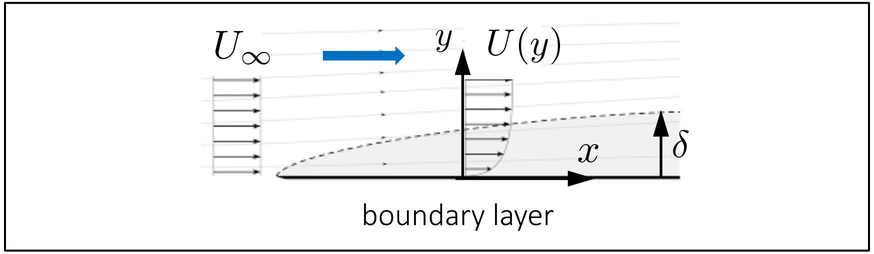
Munson video

Canonical wall-bounded flows



Couette flow





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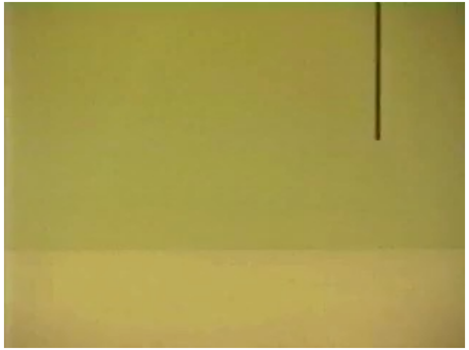
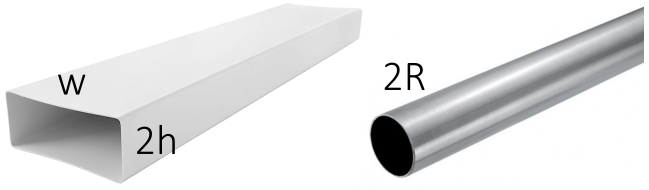


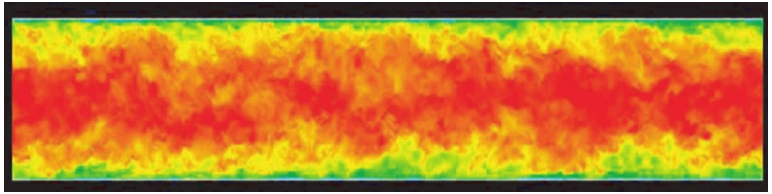
Munson video

Internal versus external flows

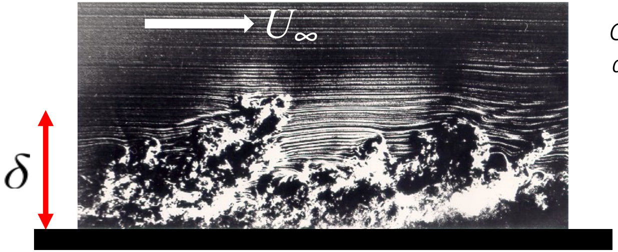
- Internal flows:
  - Non-zero turbulent kinetic energy on centerline
  - Time sharing of large structures (Dean & Bradshaw 1976)
  - Channel flow: perimeter/area = 1/h (w >> 2h)
  - Pipe flow: perimeter/area = 2/R



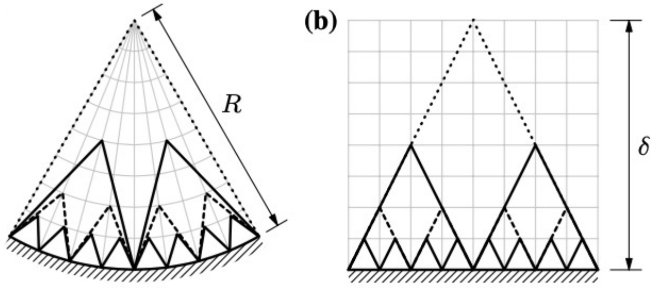
Wu & Moin (2008)



- External boundary layer flow:
  - Laminar freestream
  - Intermittency
  - Sensitive to tripping condition

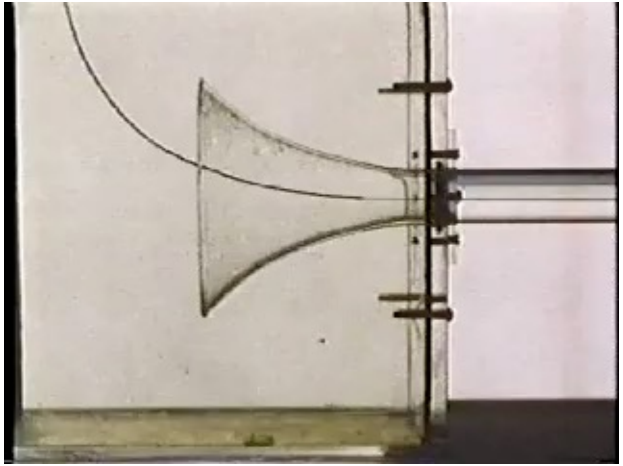


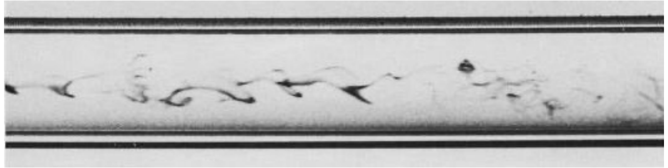
Corke, Guezennec and Nagib (1980)



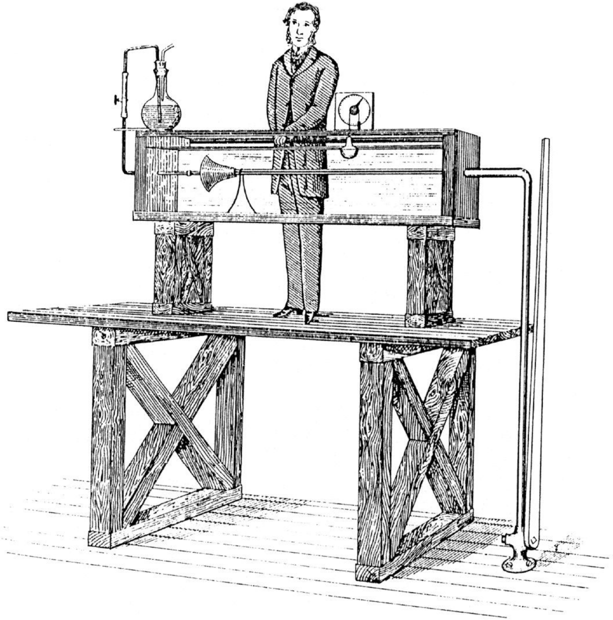
Chung et al. (2015)

Osborne Reynolds' experiment





Transition to turbulent flow



Osborne Reynolds
(1842-1916)

Munson videos

Why high Reynolds number?

- Most testing is done at low Reynolds number
- Many engineering applications are at high Reynolds number
- Most theories of turbulence only apply at high Reynolds number

$$\text{Lab:} \quad Re_\tau = 10^3, 10^4 \quad \text{Applications:} \quad Re_\tau = 10^4, 10^6$$

Windpower Engineering Vestas V112

www.newairplane.com

Turbulent flows and Reynolds number

- Reynolds number critical parameter in laminar to turbulent transition
- Turbulence continues to evolve with increasing Reynolds number

$$Re = \frac{UL}{\nu} = \frac{\text{inertia force}}{\text{viscous force}} \sim \frac{L}{\nu/U} = \frac{\text{largest eddy size}}{\text{smallest eddy size}}$$

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$$Re_\tau = \frac{\delta}{\nu/u_\tau} = \frac{\delta u_\tau}{\nu}$$

$$u_\tau = \sqrt{\tau_w/\rho}$$

(turbulence velocity scale)

inner length scale

outer length scale

$$(Re_\tau = Re^+)$$

Turbulent flows and Reynolds number

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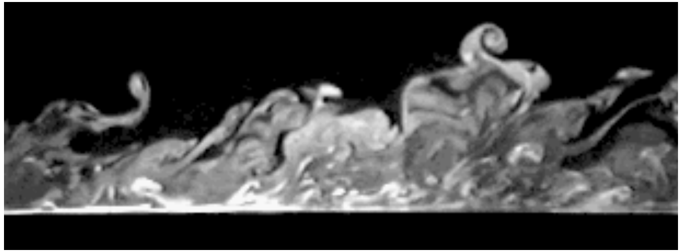
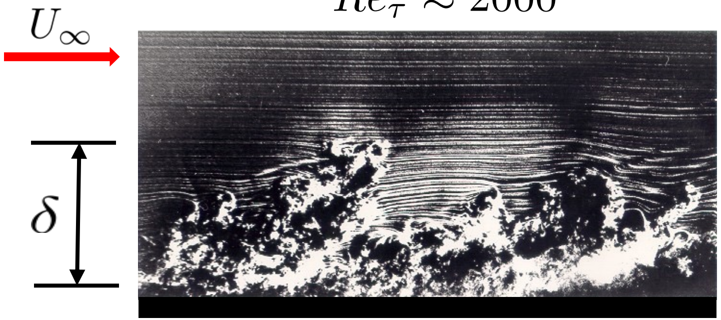
$$Re_\tau = \frac{\delta}{\nu/u_\tau} = \frac{\delta u_\tau}{\nu}$$

$Re_\tau \approx 150$



Delo, Kelso & Smits (2004)

$Re_\tau \approx 2000$



Corke, Guezennec & Nagib (1980)

The usual scaling

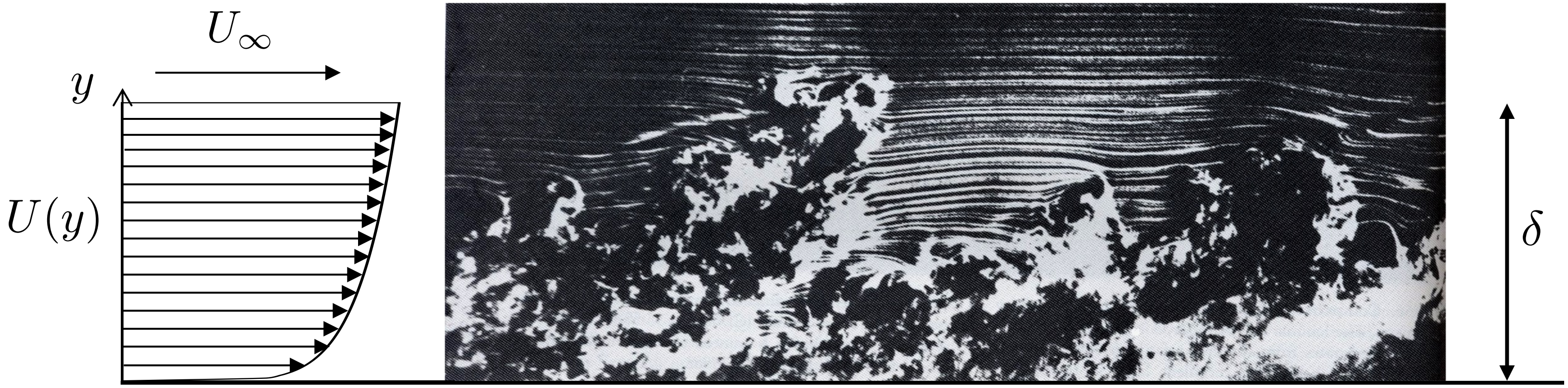
	Inner coordinates	Outer coordinates
Mean velocity	$U^+ = \dfrac{U}{u_\tau}$	$\dfrac{U_\infty - U}{u_\tau}$
Fluctuations	$\overline{u^2}^+ = \dfrac{\overline{u'^2}}{u_\tau^2}$	$\overline{u^2}^+ = \dfrac{\overline{u'^2}}{u_\tau^2}$
Wall distance	$y^+ = \dfrac{y u_\tau}{\nu}$	$\dfrac{y}{\delta}$ or $\dfrac{y}{R}$
	(viscous or inner scaling)	(outer scaling)

$$u_\tau = \sqrt{\tau_w/\rho}$$

$$Re_\tau = \frac{\delta u_\tau}{\nu}$$

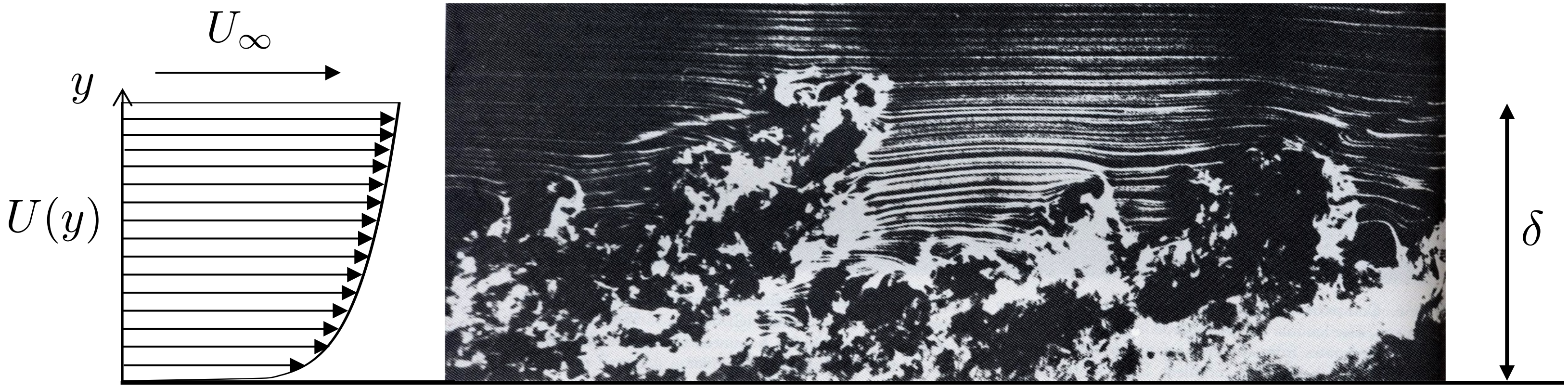
Turbulent boundary layer scaling

- Dimensional analysis: $U = f(y, \tau_w, \mu, \rho, \delta)$



Turbulent boundary layer scaling

- Dimensional analysis: $U = f(y, \tau_w, \mu, \rho, \delta)$
- Viscosity important near wall, but not important far from wall (at high Re_τ)



Turbulent boundary layer scaling

- Dimensional analysis: $$U = f(y, \tau_w, \mu, \rho, \not{\delta})$$
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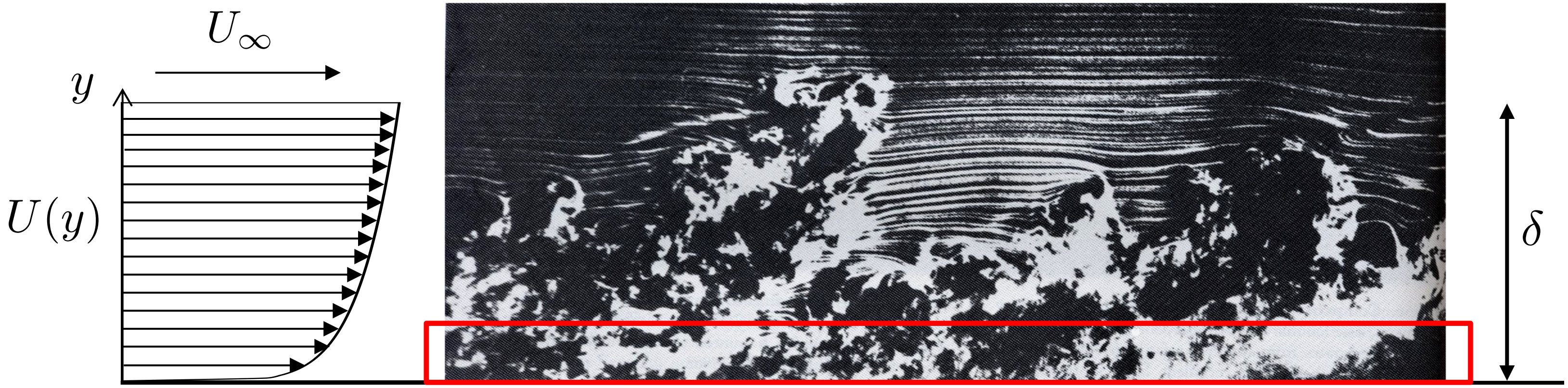
Inner scaling: $$\frac{U}{u_\tau} = f\left(\frac{y u_\tau}{\nu}\right), \quad \text{or} \quad \boxed{U^+ = f(y^+)} \qquad \left(u_\tau = \sqrt{\tau_w/\rho}\right)$$



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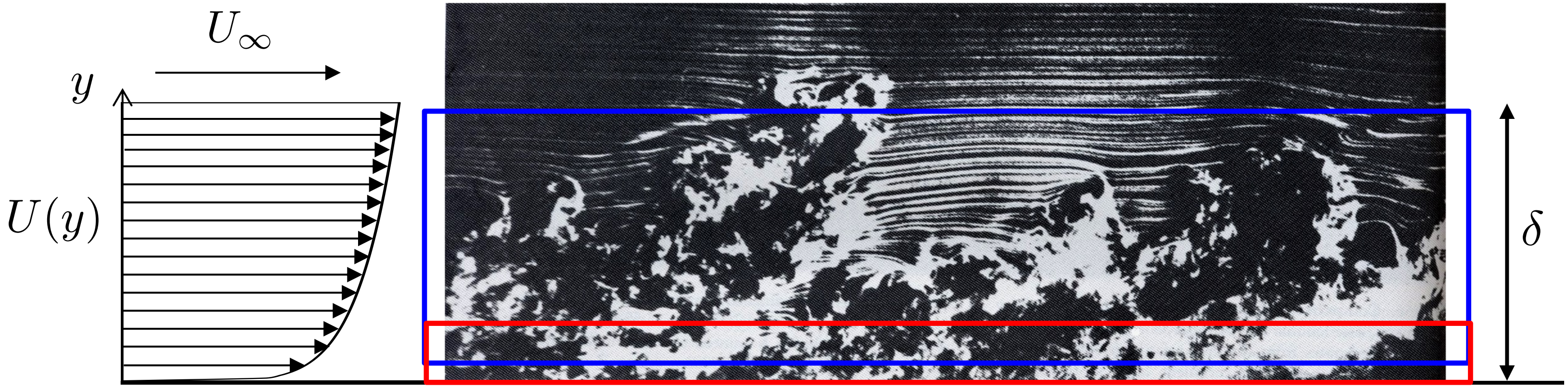
Outer scaling: $$\boxed{\frac{U_\infty - U}{u_\tau} = g\left(\frac{y}{\delta}\right)}$$



Turbulent boundary layer scaling

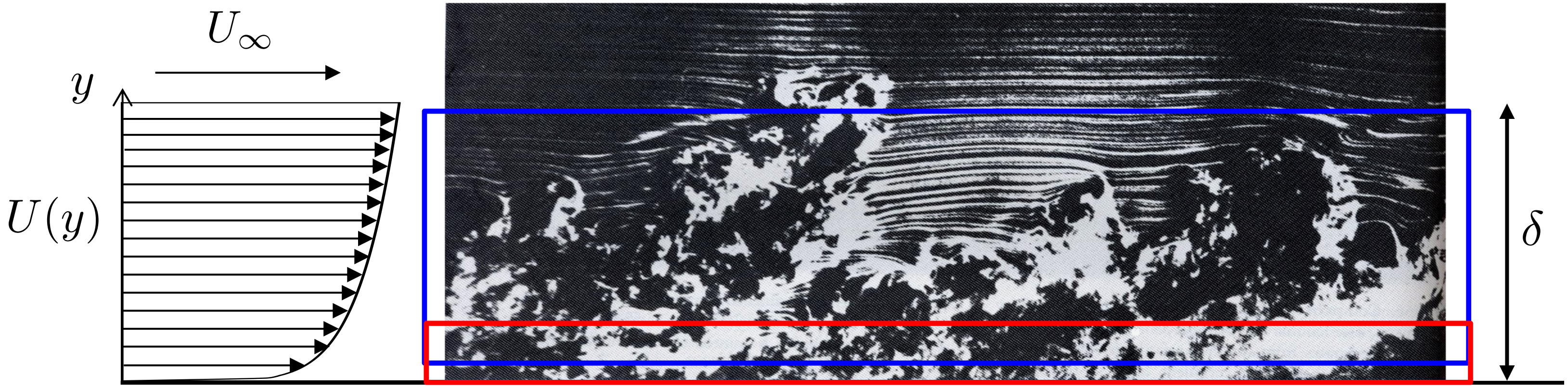
- Dimensional analysis: $U = f(y, \tau_w, \mu, \rho, \delta)$
- Viscosity important near wall, but not important far from wall (at high Re_τ)

Inner scaling: $U^+ = f(y^+)$

Outer scaling: $\dfrac{U_\infty - U}{u_\tau} = g\left(\dfrac{y}{\delta}\right)$

Match gradients in overlap region at high Re_τ →

$$U^+ = \frac{1}{\kappa} \ln y^+ + B$$

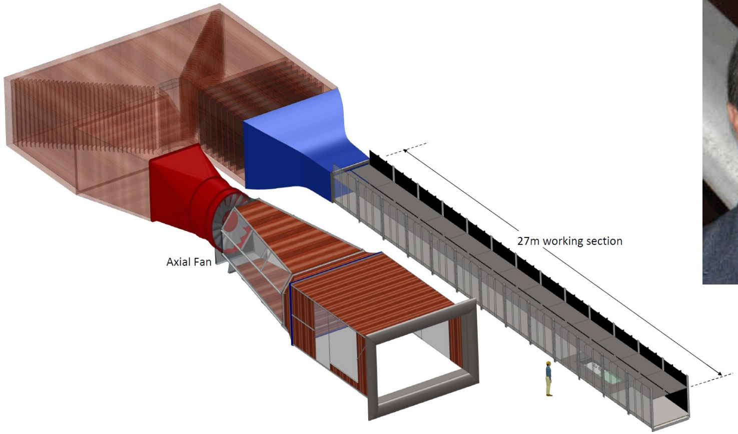


Experiments and computations

Air at pressures up to 200 bar

Princeton Superpipe: $1000 \leq Re_\tau \leq 500,000$

HRTF: $2600 \leq Re_\tau \leq 72,500$



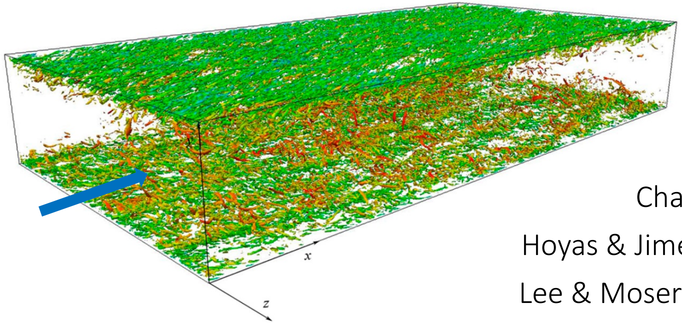
Ivan Marusic

HRNBLWT at Melbourne

$2000 \leq Re_\tau \leq 30,000$

Javier Jimenez

Robert Moser



Channel flow DNS

Hoyas & Jimenez (2003) $Re_\tau = 2000$

Lee & Moser (2015) $180 \leq Re_\tau \leq 5200$

Princeton high Reynolds number facilities

Working fluid is air at pressures up to 200 bar

Superpipe

$$31 \times 10^3 \leq Re_D \leq 35 \times 10^6$$

$$10^3 \leq Re_\tau \leq 5 \times 10^5$$

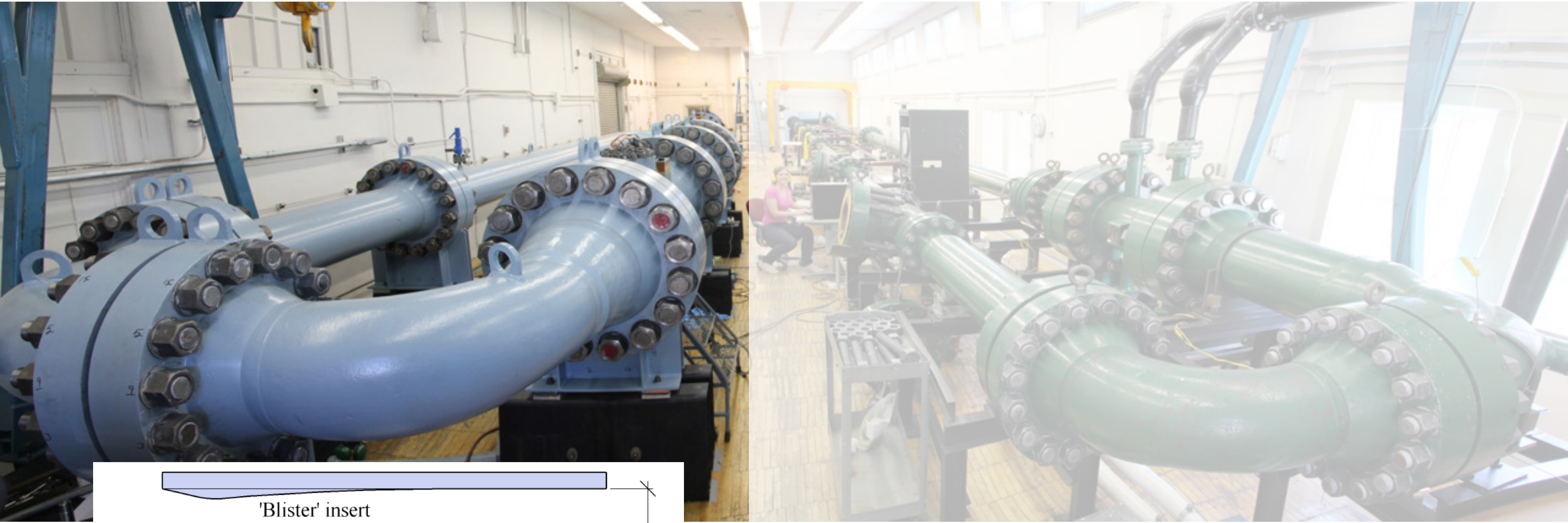
Superpipe

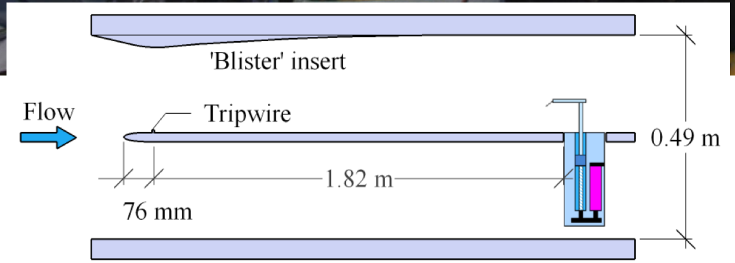
With atmospheric air at 30m/s,
pipe = 3200 m long, 1.6 m diameter

$$31 \times 10^3 \leq Re_D \leq 35 \times 10^6$$

$$10^3 \leq Re_\tau \leq 5 \times 10^5$$

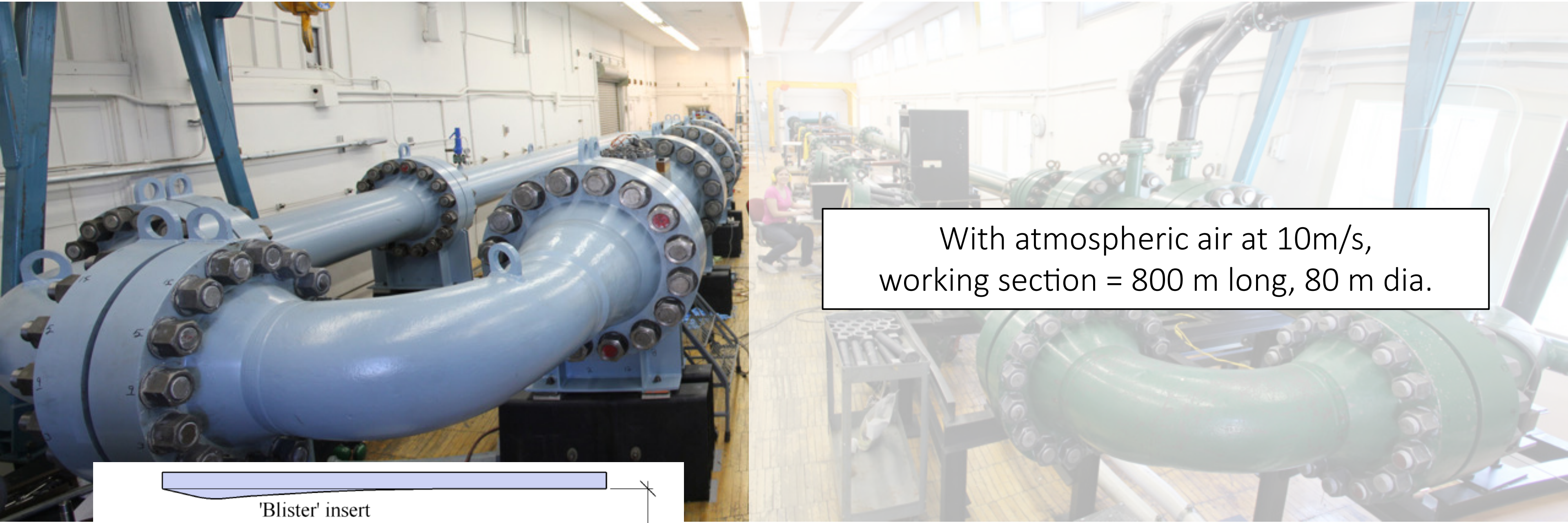
High Reynolds number test facility (HRTF)



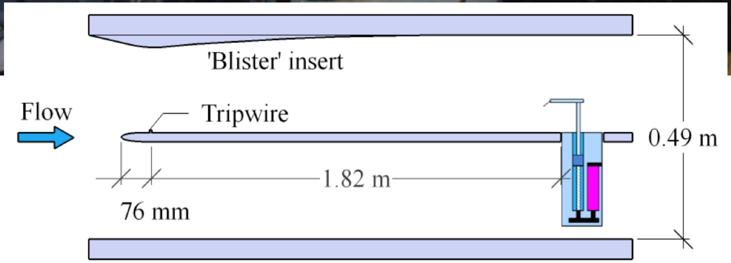


$$8400 \leq Re_\theta \leq 235,000$$
$$2600 \leq Re_\tau \leq 72,500$$

High Reynolds number test facility (HRTF)



With atmospheric air at 10m/s,
working section = 800 m long, 80 m dia.



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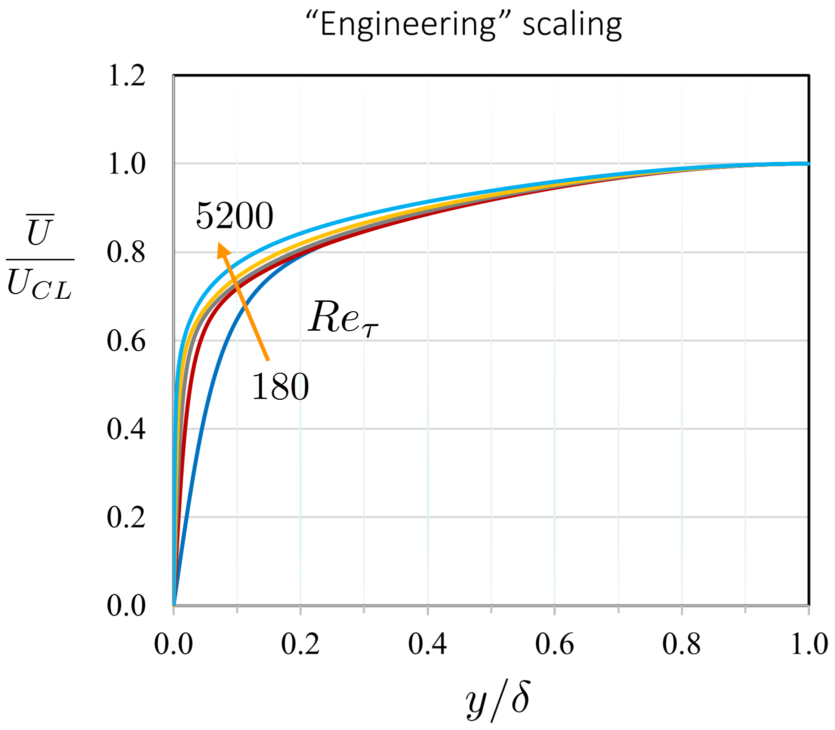
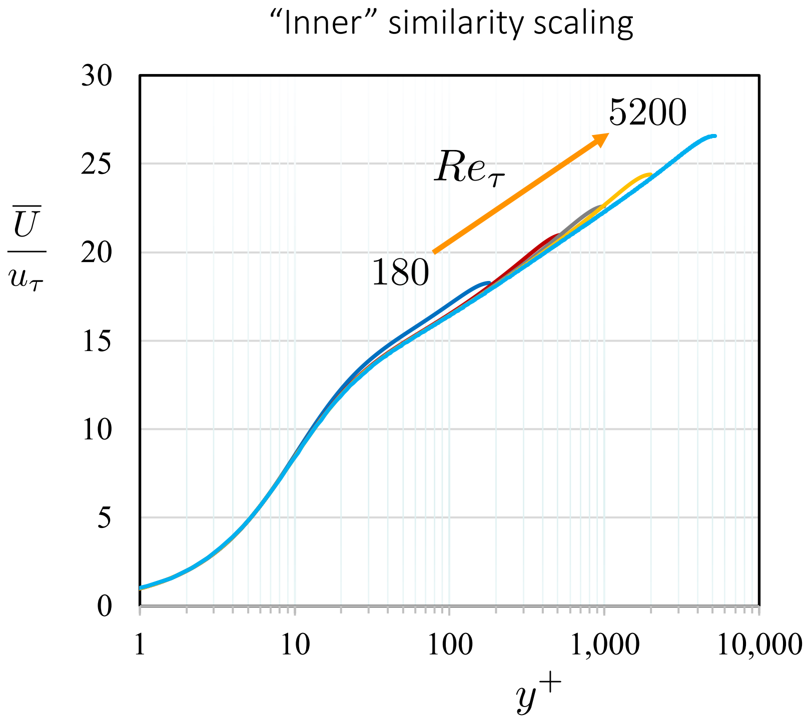
Mean velocity: channel flow DNS Re_τ = 180 - 5200





Data from Lee & Moser (2015)

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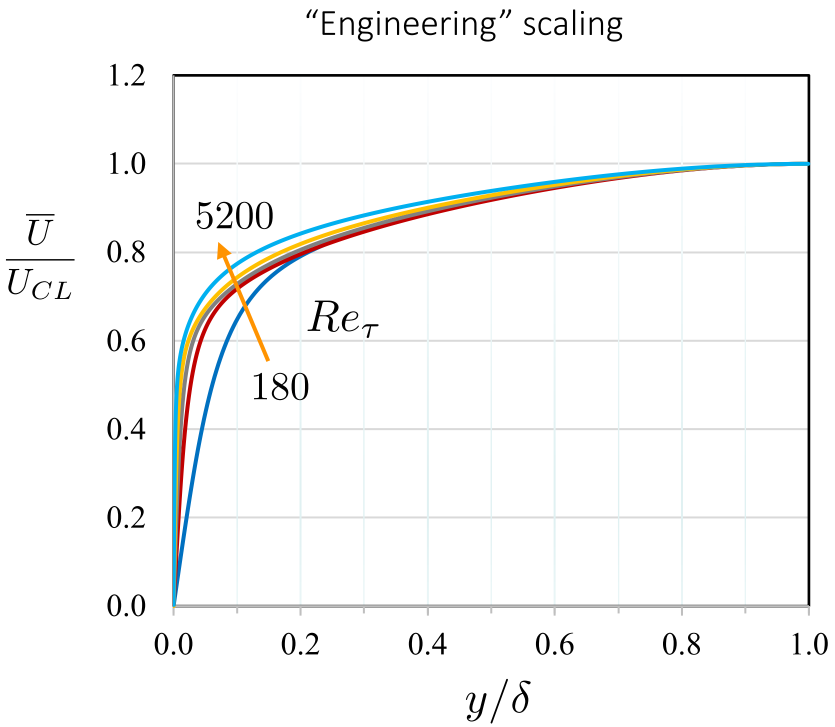

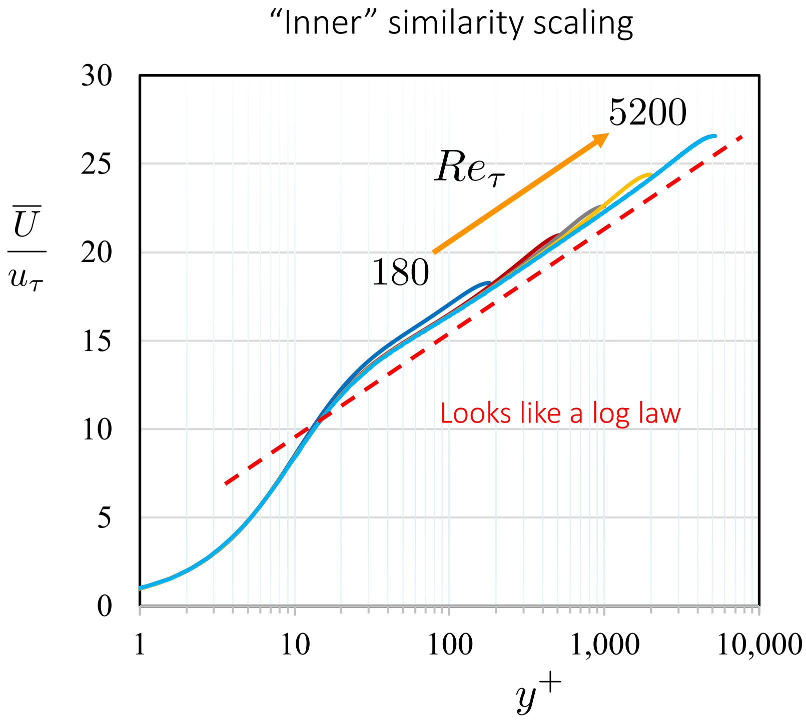

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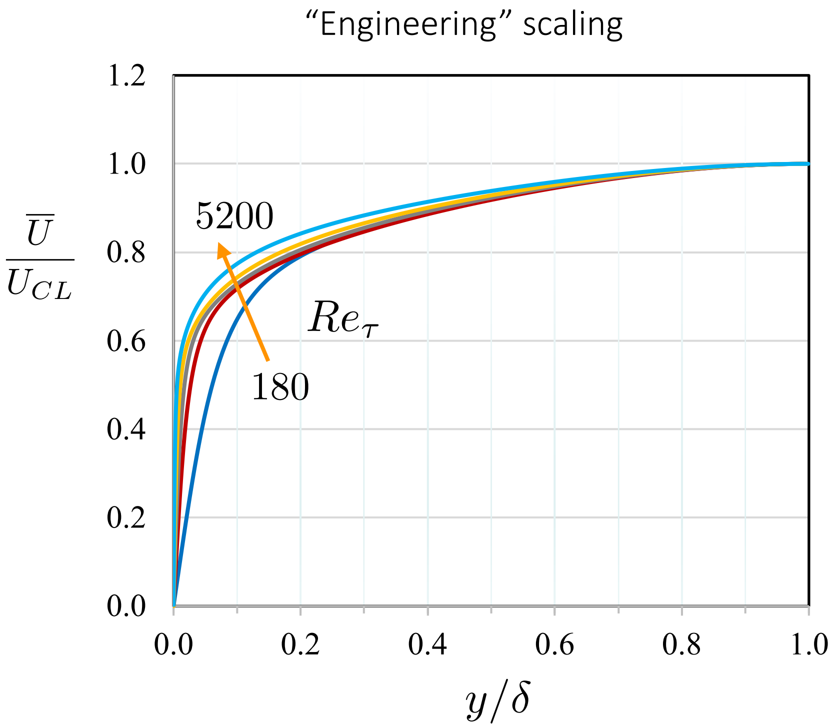

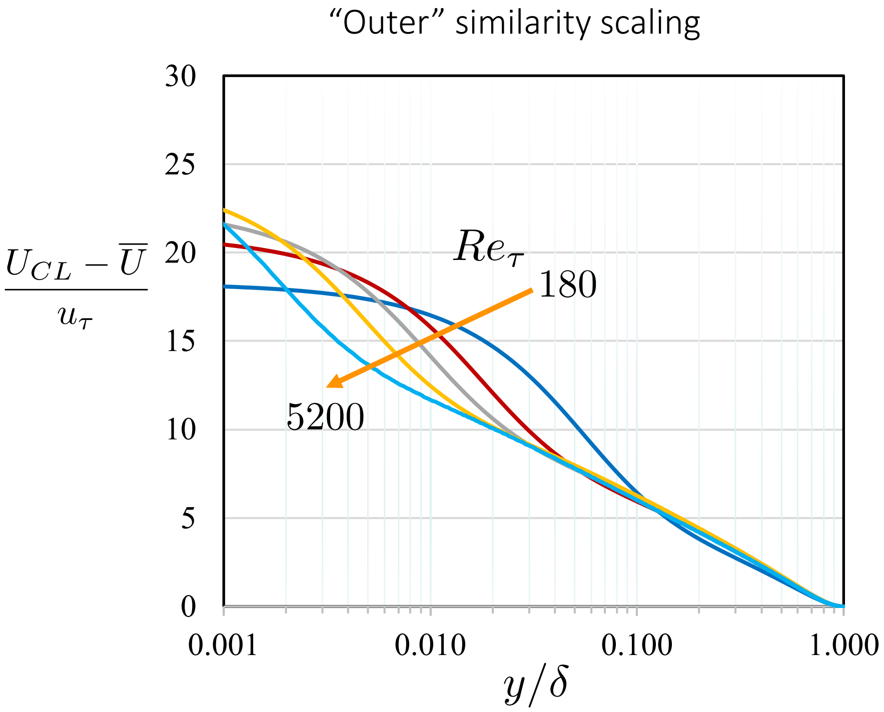

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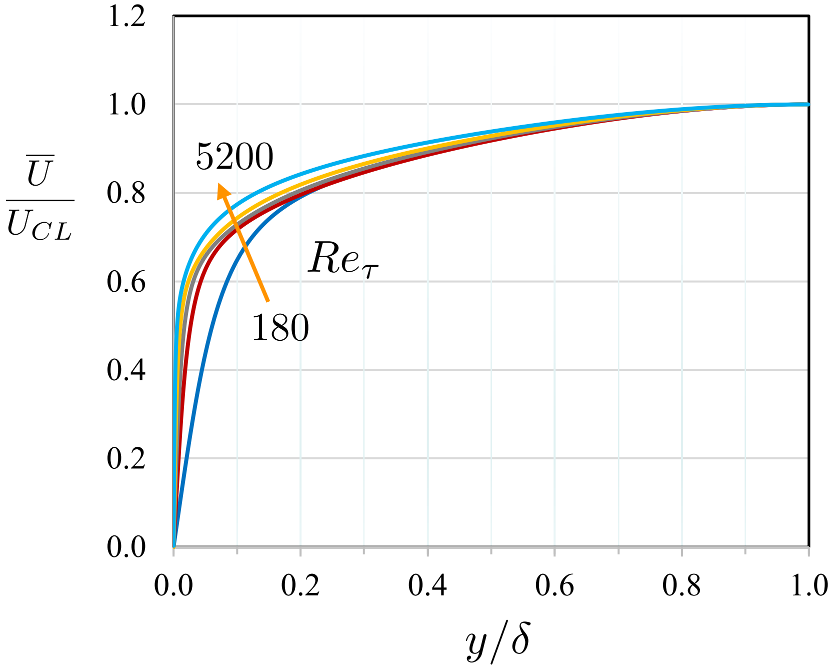
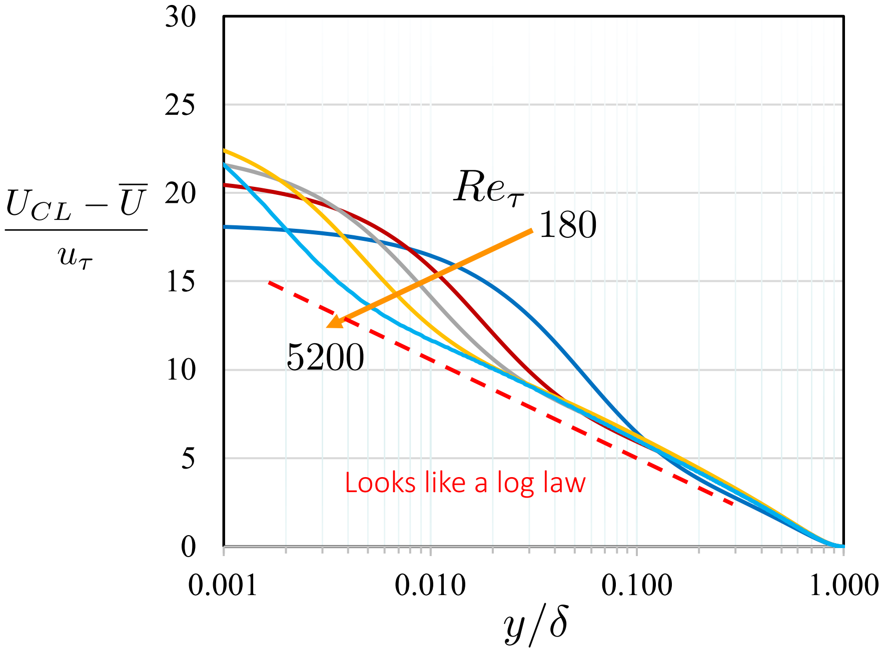
Mean velocity: channel flow DNS Re_τ = 180 - 5200

“Engineering” scaling



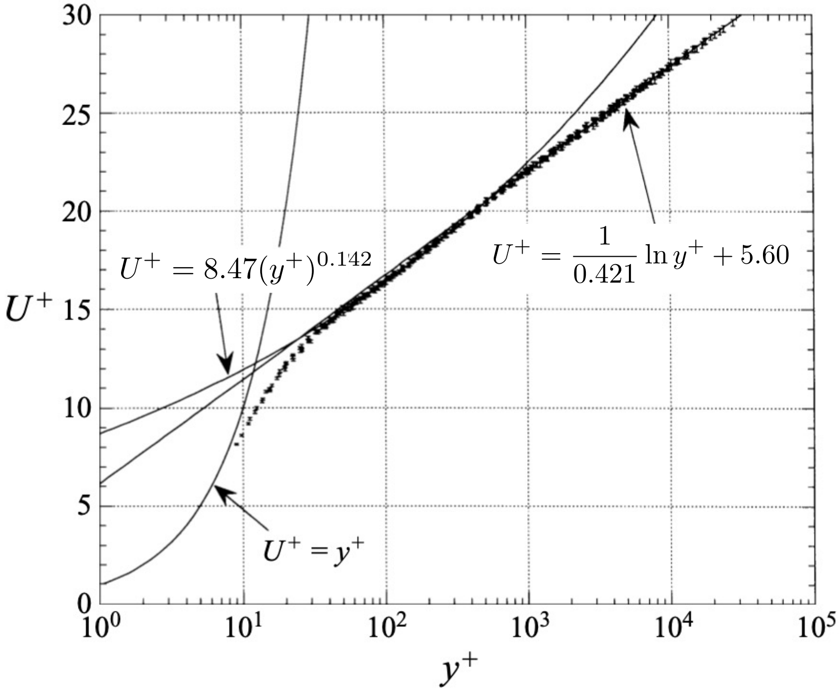
“Outer” similarity scaling



Data from Lee & Moser (2015)

Inner scaling revisited at high Reynolds number

- Log-law was derived by matching gradients of velocity in the overlap region
- However, if we match velocity gradients and magnitudes in the overlap region, a power law can be derived: $U^+ = C_1 \left(y^+\right)^\gamma$



Beverley McKeon

Mark Zagarola

Margit Vallikivi

Zagarola & Smits (1998)
McKeon, Li, Jiang, Morrison & Smits (2004)
Bailey, Vallikivi, Hultmark & Smits (2014)

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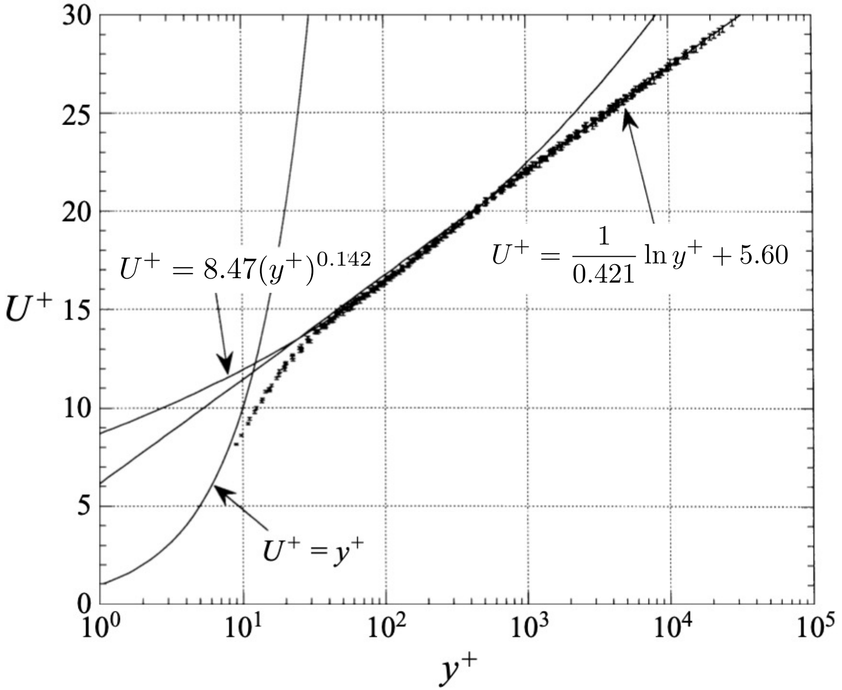
- Experiments reveal that a power law joins the viscous sublayer to the log-law
- The log-law begins at y+ = 600
- It ends at y/δ = 0.12
- Log-law only appears for Re_τ > 10,000 (one octave), or 50,000 (one decade)



Zagarola & Smits (1998)
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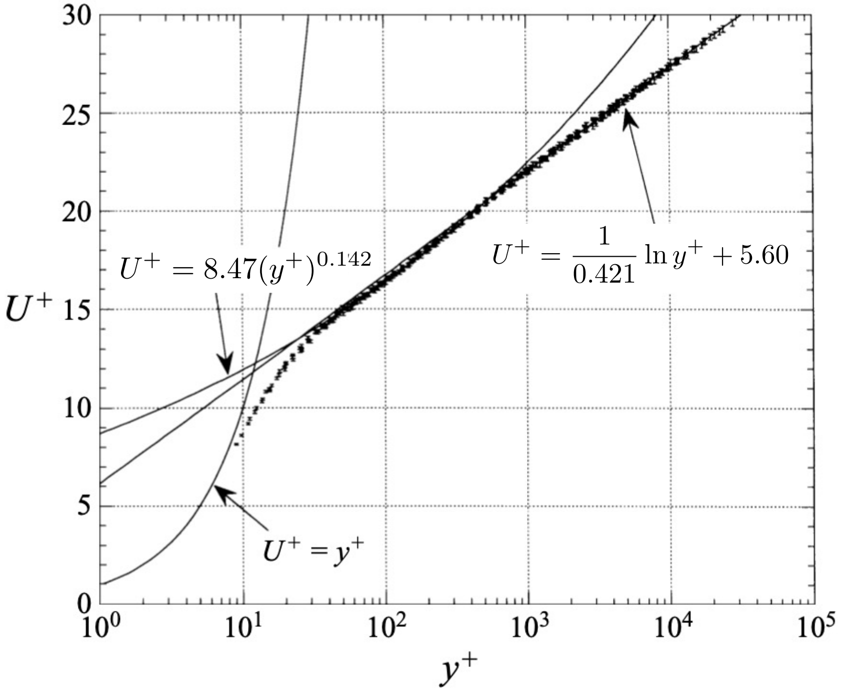
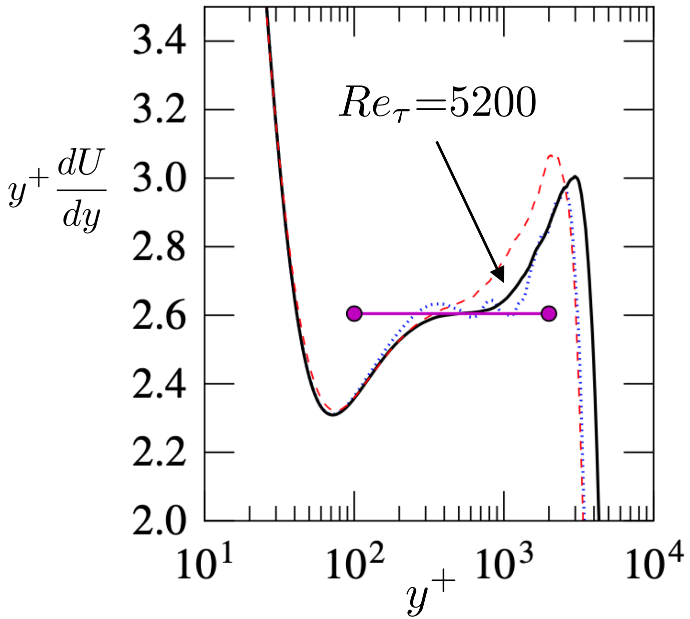
- Log-law only appears for Re_τ > 10,000 (one octave), or 50,000 (one decade)
- What about κ?
- Best estimate for pipe flow $\kappa = 0.40 \pm 0.02$
- Other people find more precise values
- Can DNS help?



Zagarola & Smits (1998)
McKeon, Li, Jiang, Morrison & Smits (2004)
Bailey, Vallikivi, Hultmark & Smits (2014)

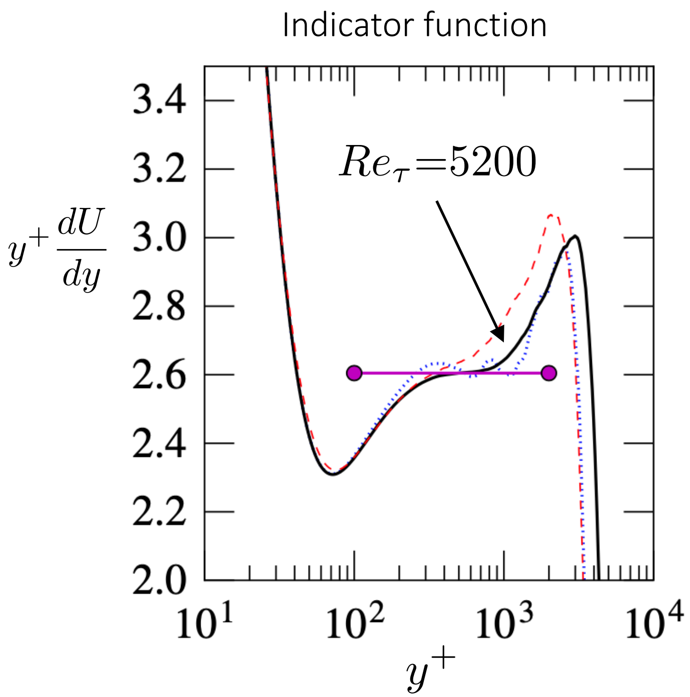
Channel flow DNS (max Re_τ = 5200)

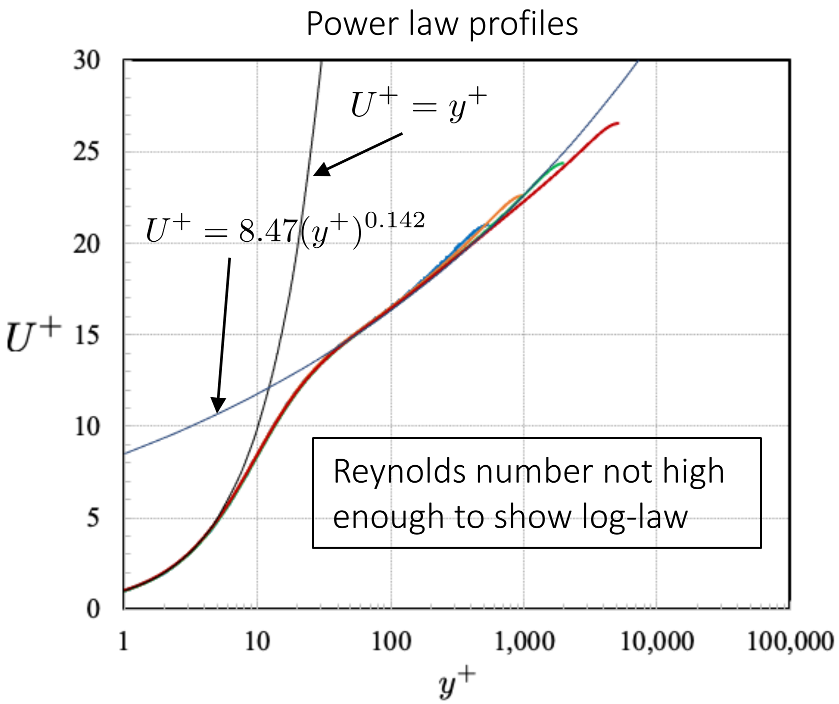
Indicator function



DNS by Lee & Moser (2015)

Channel flow DNS (max Re_τ = 5200)





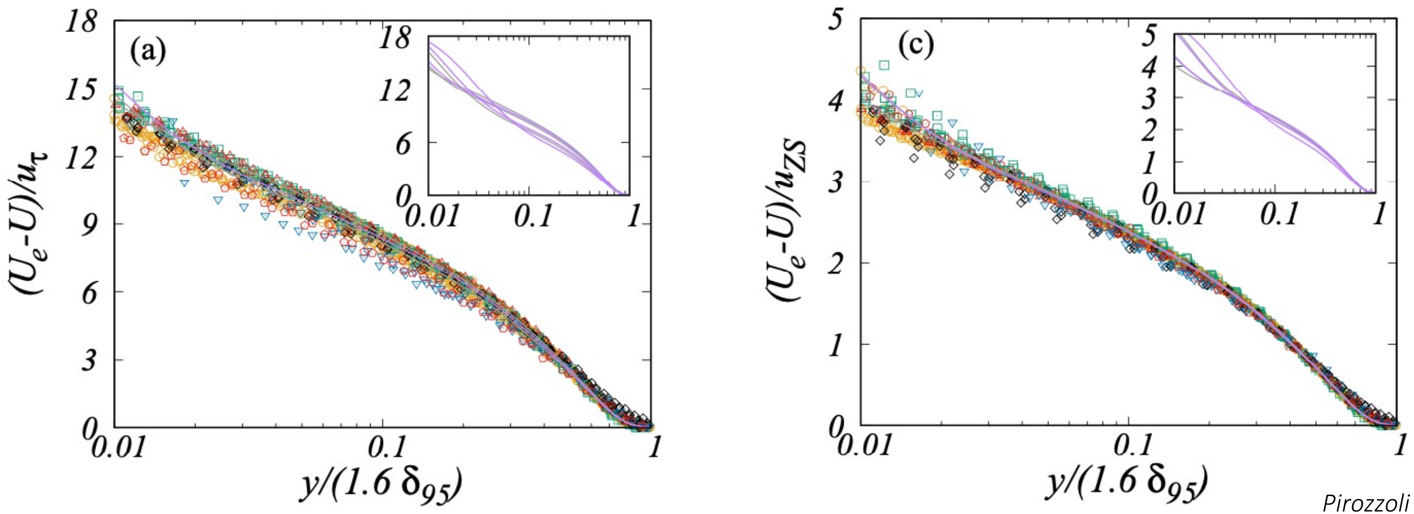
$$\kappa = 0.40 \pm 0.02$$

(will probably need DNS at much higher Reynolds number to be more precise)

DNS by Lee & Moser (2015)

Outer scaling revisited

- Dimensional analysis: $U = f(y, u_\tau, \mu, \rho, \delta)$ $\left(u_\tau = \sqrt{\tau_w/\rho}\right)$
- Assumes one velocity scale for inner and outer regions
- Experiments suggest both an inner scale (u_τ) and an outer scale (u_{ZS})
- Propose $u_{ZS} = (\delta^*/\delta)\, U_\infty$ (Zagarola & Smits 1997; 1998)
- In the outer region scale u_{ZS} works better than u_τ at lower Reynolds numbers



Pirozzoli & Smits (2023)

The velocity profile and the friction factor

By integrating the velocity profile, we can obtain a friction factor/Reynolds number relationship:

$$\lambda \equiv \frac{-\frac{dp}{dx} D}{\frac{1}{2}\rho \overline{U}^2} = \frac{4\tau_w}{\frac{1}{2}\rho \overline{U}^2} = 8\left(\frac{u_\tau}{\overline{U}}\right)^2$$

That is, $$\sqrt{\frac{8}{\lambda}} = \frac{\overline{U}}{u_\tau} = 2\int_0^1 U^+ \left(1 - \frac{y^+}{R^+}\right) d\frac{y^+}{R^+}$$

With $$U^+ = \frac{1}{\kappa}\ln y^+ + B$$

$$\Rightarrow \quad \frac{1}{\sqrt{\lambda}} = C_1 \log(Re_D\sqrt{\lambda}) + C_2 + C_3 + C_4(Re_D)$$

outer layer (wake) deviation (C_3)

inner layer deviation ($C_4(Re_D)$)

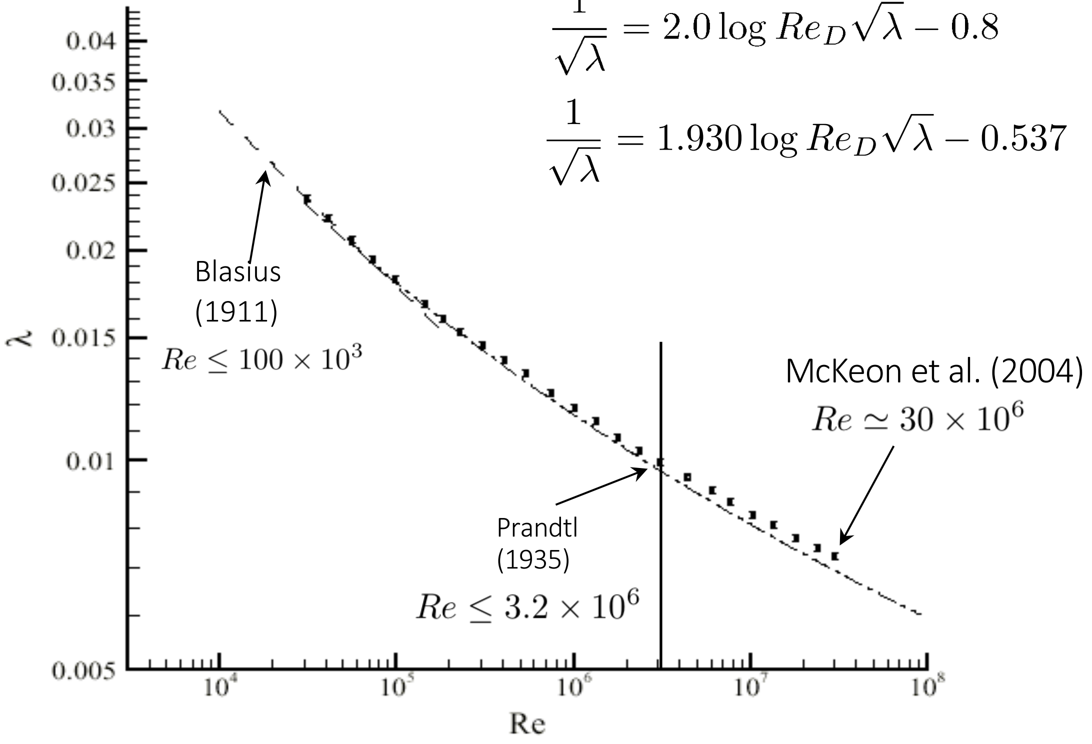
$$\boxed{\frac{1}{\sqrt{\lambda}} = 2.0\log(Re_D\sqrt{\lambda}) - 0.8}$$

Prandtl smooth pipe

Superpipe results

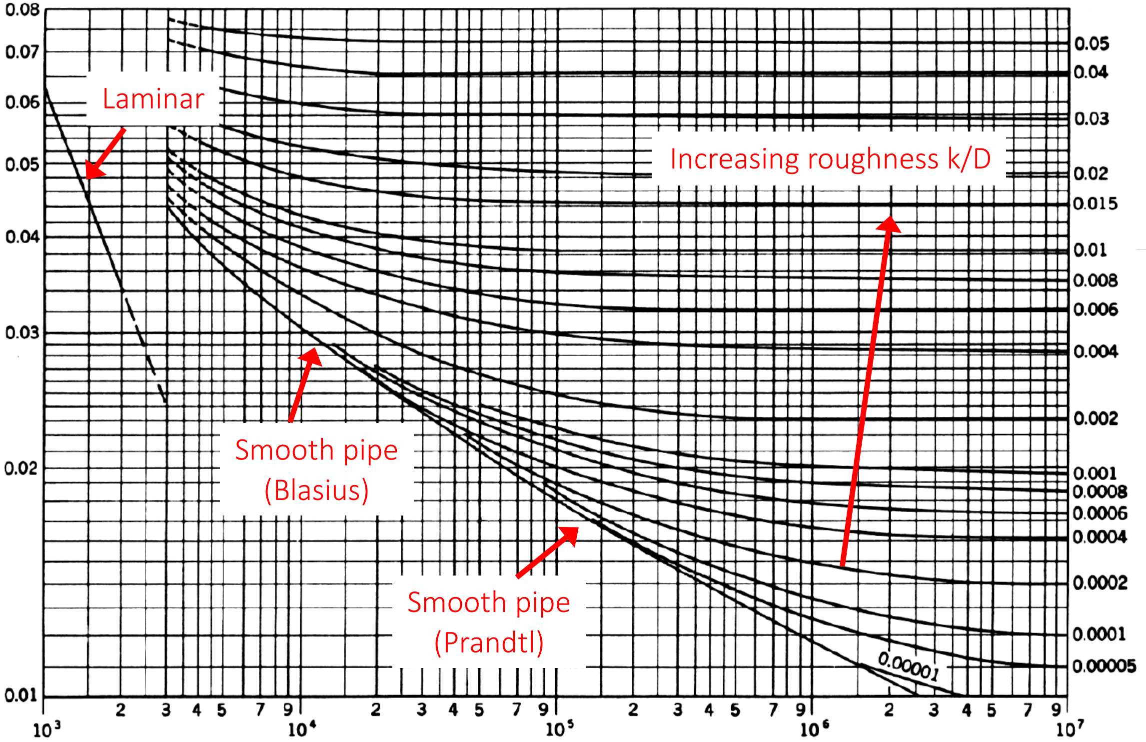
$$\frac{1}{\sqrt{\lambda}} = 2.0 \log Re_D \sqrt{\lambda} - 0.8$$ Prandtl (1935)

$$\frac{1}{\sqrt{\lambda}} = 1.930 \log Re_D \sqrt{\lambda} - 0.537$$ McKeon et. al. (2004)



Pipe flow friction: the Moody Diagram

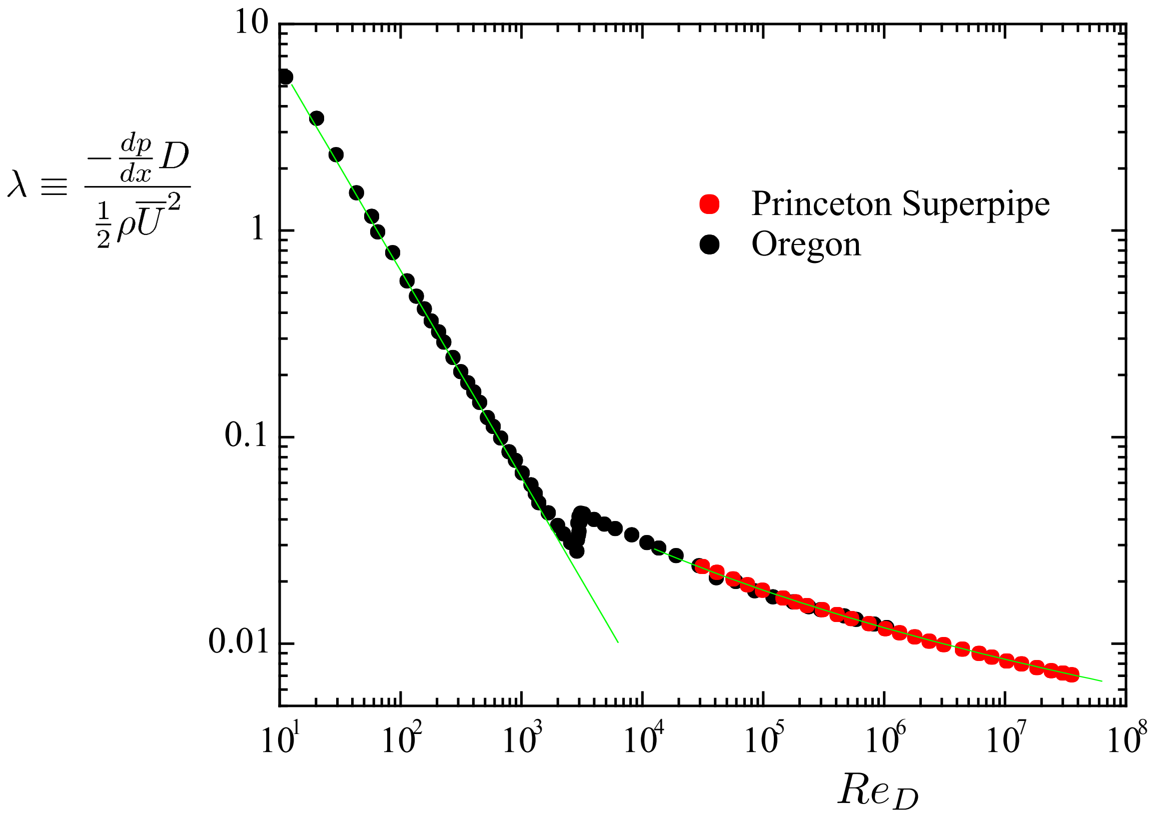
$$\lambda \equiv \frac{-\frac{dp}{dx} D}{\frac{1}{2}\rho \overline{U}^2}$$



k/D

$Re_D = \overline{U}D/\nu$

Two complementary experiments



McKeon et al. (2004)

Turbulent stress scaling

- Dimensional analysis: $\overline{u_i u_j} = f(y, u_\tau, \mu, \rho, \delta)$ $\left(u_\tau = \sqrt{\tau_w/\rho}\right)$
- Using the inner/outer overlap argument:

Match amplitudes $\overline{u_i u_j} = \text{constant}$

Match gradients $\overline{u_i u_j} = B_i - A_i \ln(y/\delta)$

Match gradients and amplitudes $\overline{u_i u_j} = C_i (y/\delta)^{\gamma_i}$

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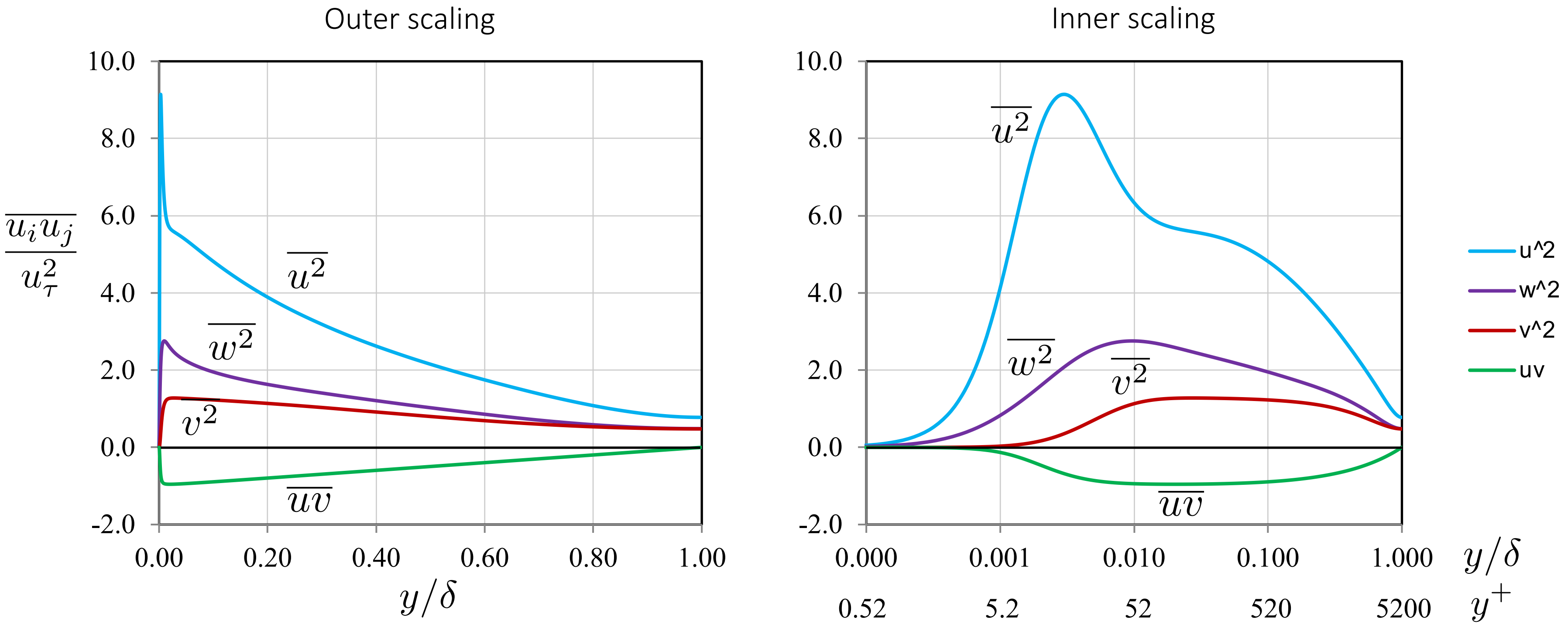
$$\left.\begin{array}{lrl} \text{Match amplitudes} & \overline{u_i u_j} = & \text{constant} \\ \text{Match gradients} & \overline{u_i u_j} = & B_i - A_i \ln(y/\delta) \\ \text{Match gradients and amplitudes} & \overline{u_i u_j} = & C_i (y/\delta)^{\gamma_i} \end{array}\right\} ?$$

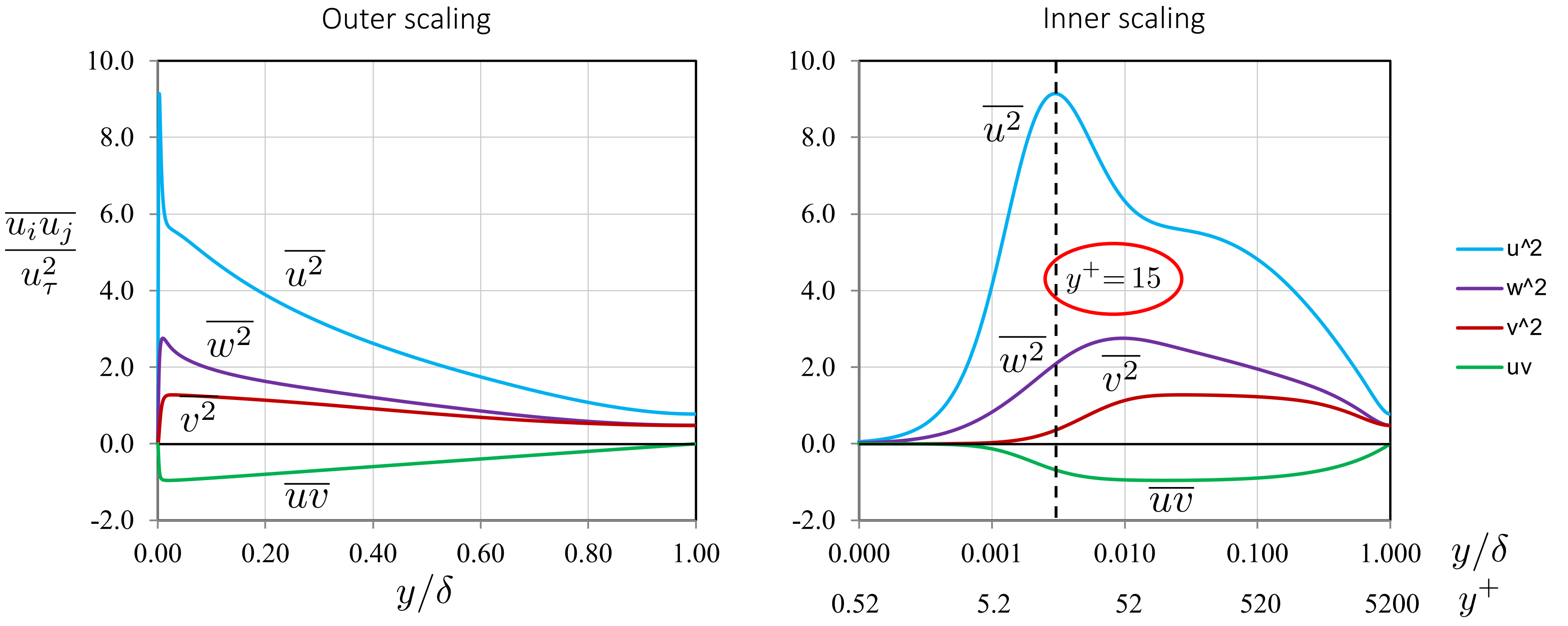
- As we shall see, different components of the stress tensor follow different scaling in the overlap region
- For example, $\overline{u^2}/u_\tau^2$ follows a logarithmic variation, while $\overline{v^2}/u_\tau^2$ is a constant
- No power law behavior has been seen

Turbulent stress levels: channel flow DNS Re_τ = 5200



Data from Lee & Moser (2015)

Turbulent stress levels: channel flow DNS $Re_\tau = 5200$



Data from Lee & Moser (2015)

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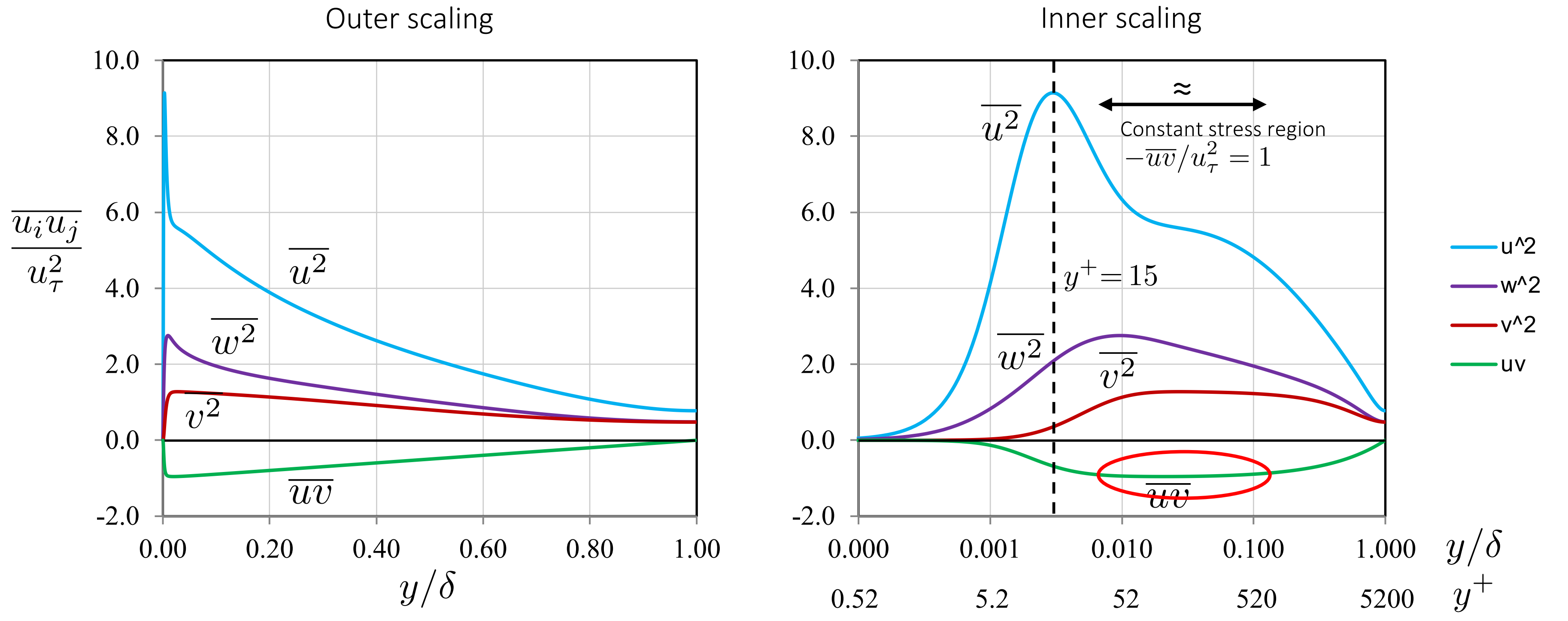
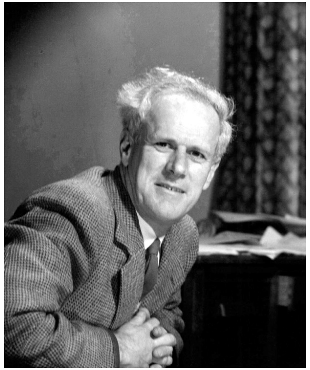
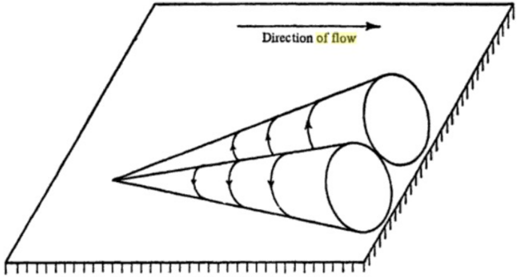


Data from Lee & Moser (2015)

Scaling the turbulence: the Attached Eddy Hypothesis

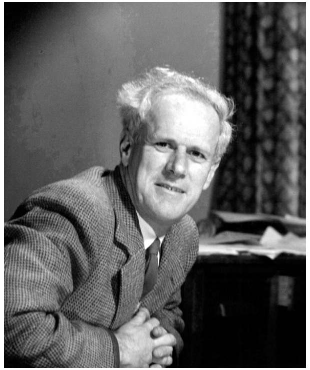


Townsend: "In other words, the velocity fields of the main eddies, regarded as persistent, organized flow patterns, extend to the wall and, in a sense, they are attached to the wall."



Townsend (1976)

Scaling the turbulence: the Attached Eddy Hypothesis



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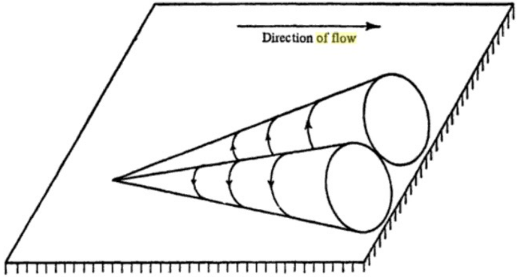
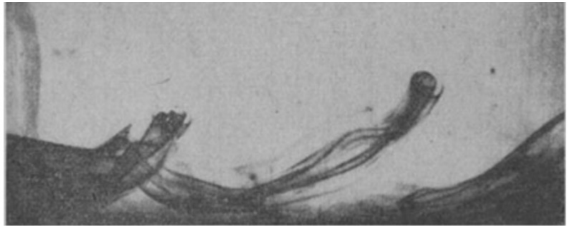
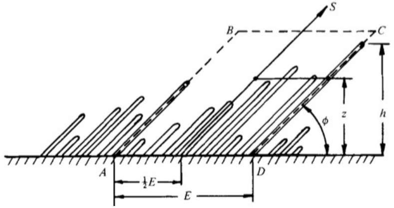


Townsend (1976)

<u>Perry</u>: In this theory, wall turbulence is considered to consist of a 'forest' of randomly positioned horseshoe, hairpin or Λ-shaped vortices that lean in the streamwise direction and have their legs extending to the wall.



Theodorsen (1952)



Perry & Chong (1982)

Townsend/Perry Attached Eddy Model

- The model is inviscid (high Reynolds number), and considers a superposition of geometrically self-similar, attached eddies
- The eddies cover a wide range of scales, but each scale is proportional to the distance from the wall
- The eddies have the same characteristic velocity scale
- At high enough Reynolds number, the model is designed to give $-\overline{uv}/u_\tau^2 = 1$
- Model applies in the constant stress (logarithmic) region

At high Reynolds number, the model then predicts:

$$\frac{\overline{u^2}}{u_\tau^2} = B_1 - A_1 \ln\left(\frac{y}{\delta}\right)$$

$$\frac{\overline{v^2}}{u_\tau^2} = A_2$$

$$\frac{\overline{w^2}}{u_\tau^2} = B_3 - A_3 \ln\left(\frac{y}{\delta}\right)$$

Need accurate measurements at high Reynolds number

Townsend/Perry Attached Eddy Model

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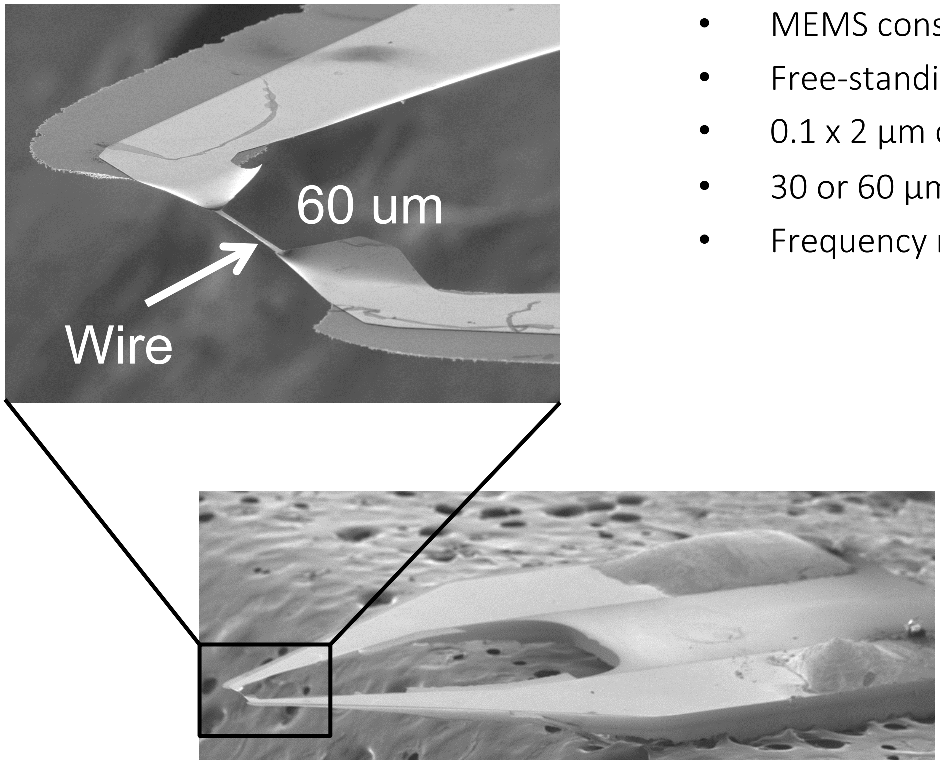
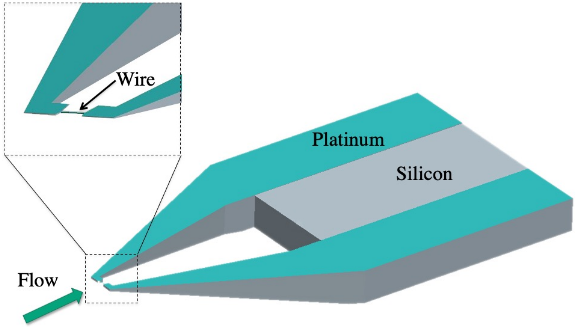
Nano-Scale Thermal Anemometry Probe (NSTAP)



- MEMS construction
- Free-standing Pt ribbon
- 0.1 x 2 µm cross-section
- 30 or 60 µm sensing length
- Frequency response > 150kHz

Marcus Hultmark



Bailey et al. (2010)
Vallikivi , Bailey, Hultmark & Smits (2011)
Vallikivi & Smits (2014)
Hutchins, Monty, Hultmark, Smits (2015)

Superpipe turbulence data (NSTAP)

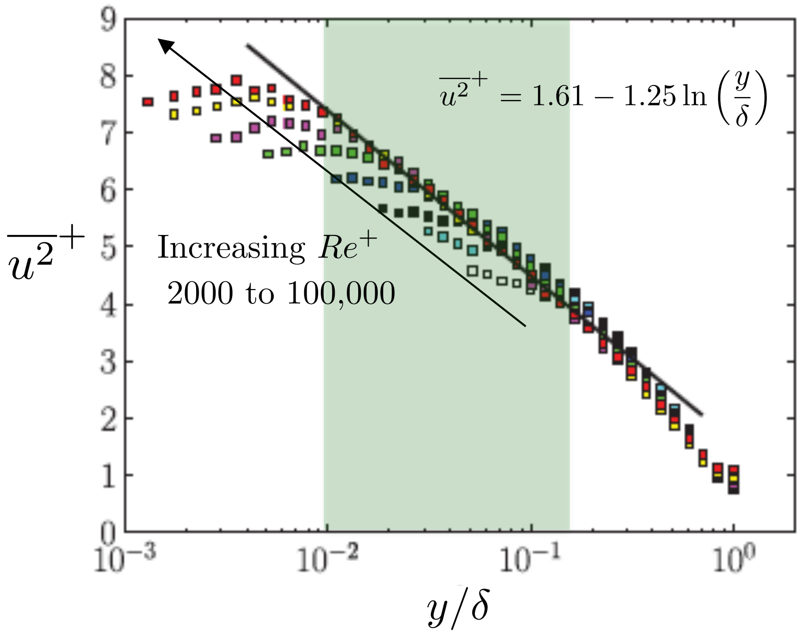
- NSTAP measurements established, unambiguously for the first time, the log law in the turbulence

$$\overline{u^2}^+ = 1.61 - 1.25 \ln\left(\frac{y}{\delta}\right)$$

- This is a key result in the Attached Eddy Model of Townsend/Perry

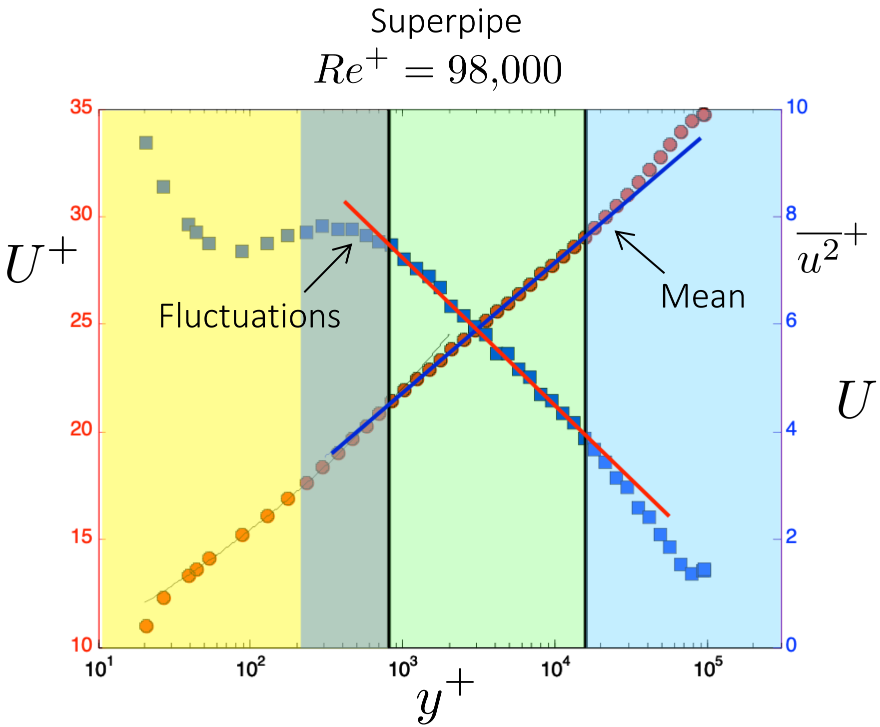
- Holds for pipes and boundary layers, with the same slope (A_1=-1.25)

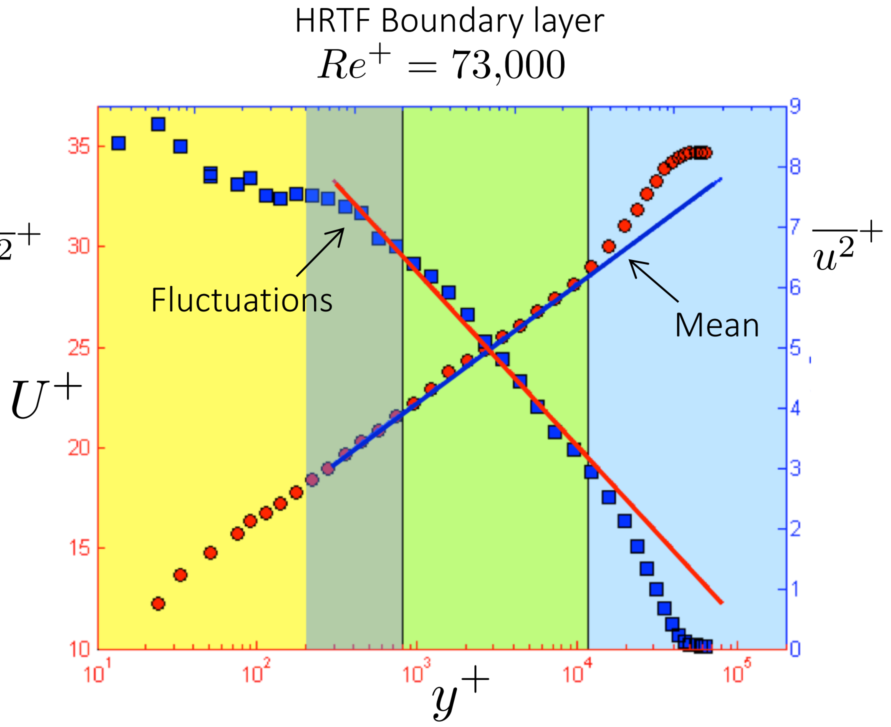
Data for $y^+ > 100$



Hultmark , Vallikivi, Bailey & Smits (2012)
Marusic, Monty, Hultmark & Smits (2013)
Vallikivi, Hultmark & Smits (2015)

Boundary layer vs. pipe flow

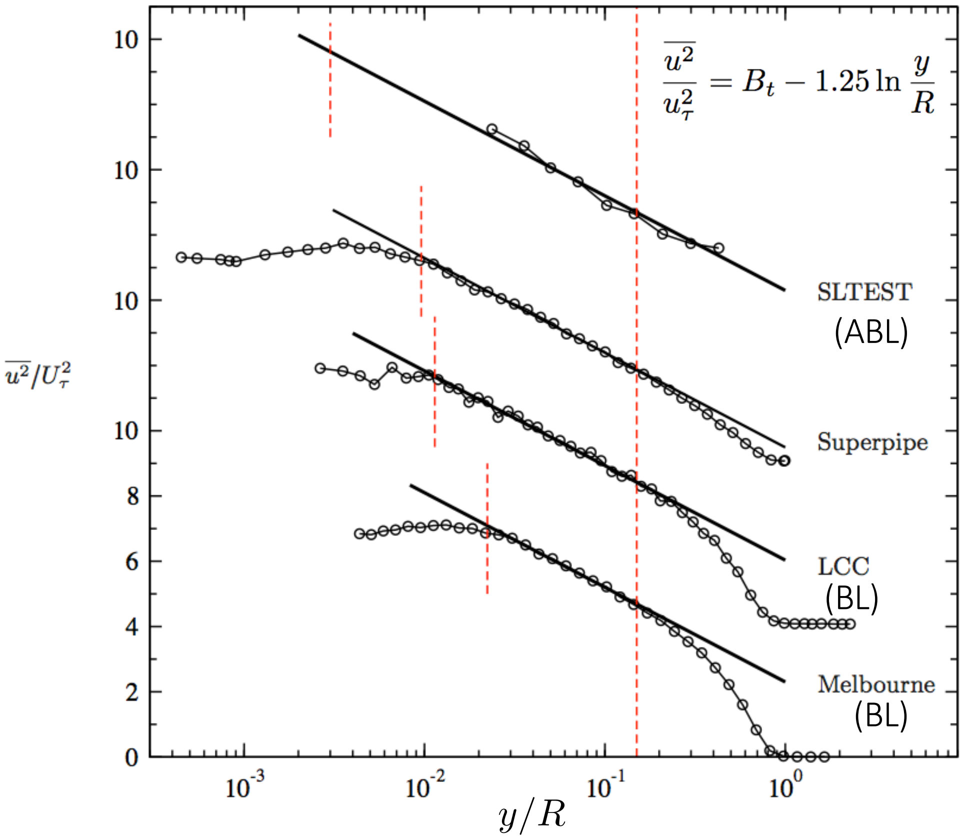




Hultmark, Vallikivi, Bailey & Smits (2013)

Vallikivi, Hultmark & Smits (2015)

A universal log law for turbulence?

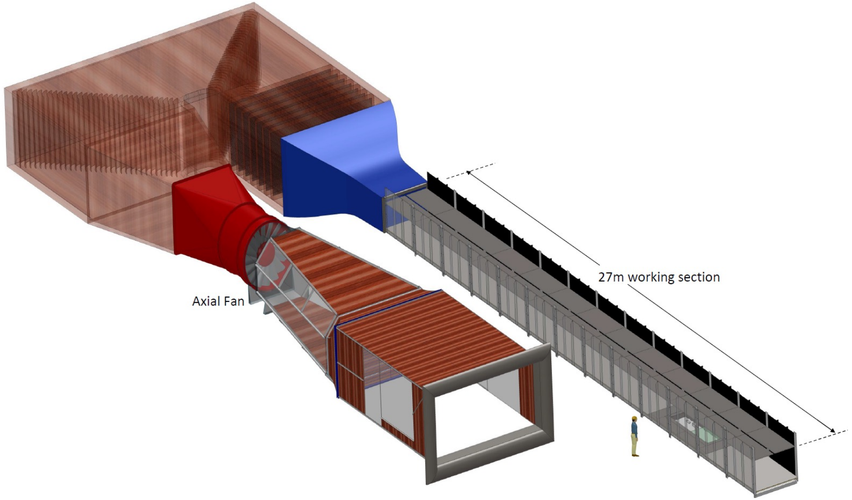


Marusic, Monty, Hultmark & Smits (2012)

NSTAP measurements in the Melbourne tunnel

Thick boundary layer, small $\ell^+ = \ell/\eta_v$

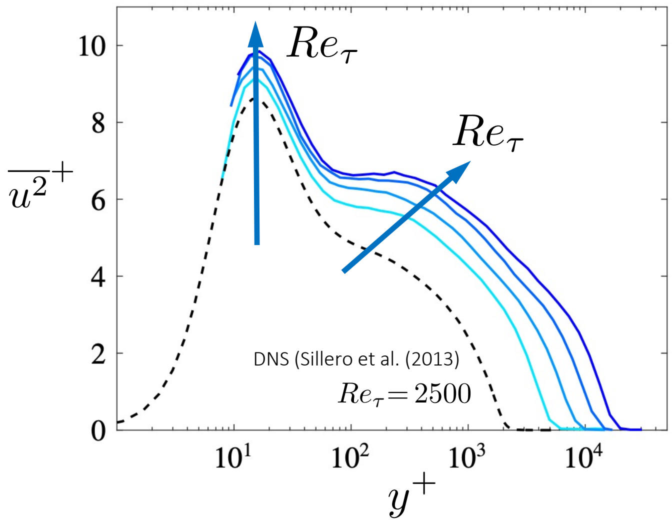
$$2.4 \leq \ell^+ \leq 3.5$$



Samie , Marusic, Hutchins, Fan, Fu, Hultmark & Smits (2018)

NSTAP measurements in the Melbourne tunnel

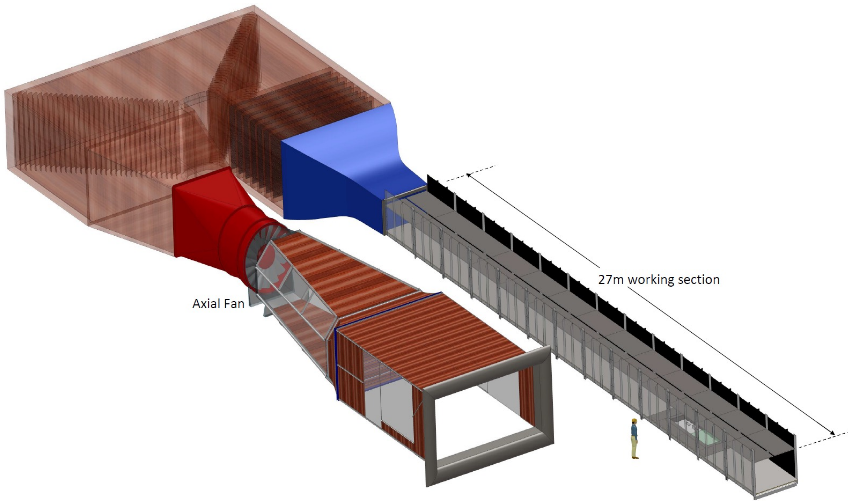
$Re_\tau = 6123,\ 10100,\ 14680,\ 19680$



The "inner" peak near $y^+=15$ grows with Reynolds number

Thick boundary layer, small $\ell^+ = \ell/\eta_v$

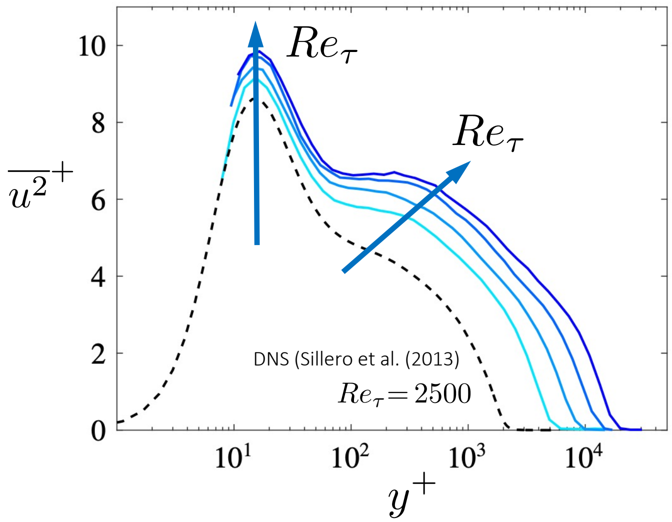
$$2.4 \leq \ell^+ \leq 3.5$$

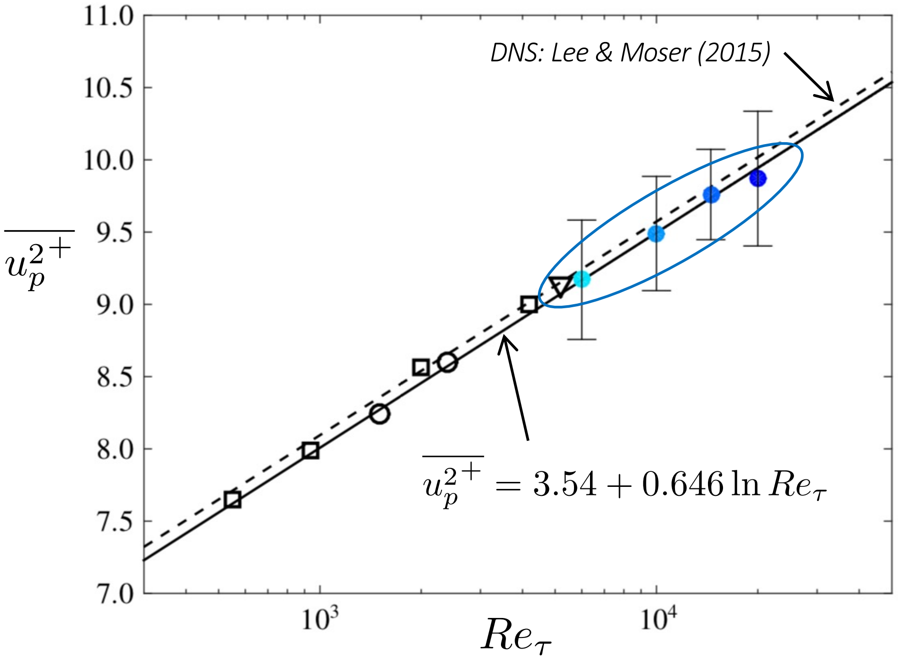


Samie , Marusic, Hutchins, Fan, Fu, Hultmark & Smits (2018)

Growth of the inner peak

$Re_\tau = 6123,\ 10100,\ 14680,\ 19680$





Samie , Marusic, Hutchins, Fan, Fu, Hultmark & Smits (2018)

Why does the inner peak grow with Reynolds number?

How far can we get analytically?

How about a Taylor series expansion for small y^+:

$$(U+u)^+ = a_1 + b_1 y^+ + c_1 {y^+}^2 + d_1 {y^+}^3 + O({y^+}^4)$$

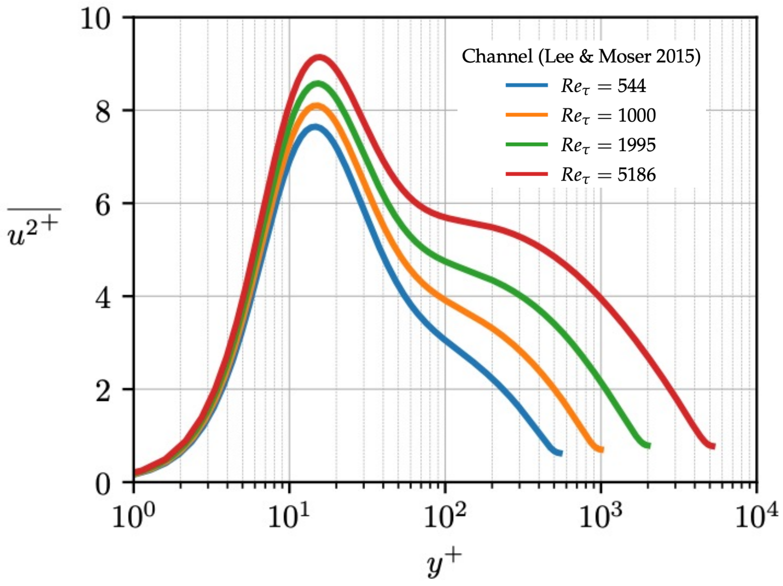
$$\Rightarrow \quad \overline{u^2}^+ = f_{u^2}\, {y^+}^2 \quad (y^+ \to 0)$$

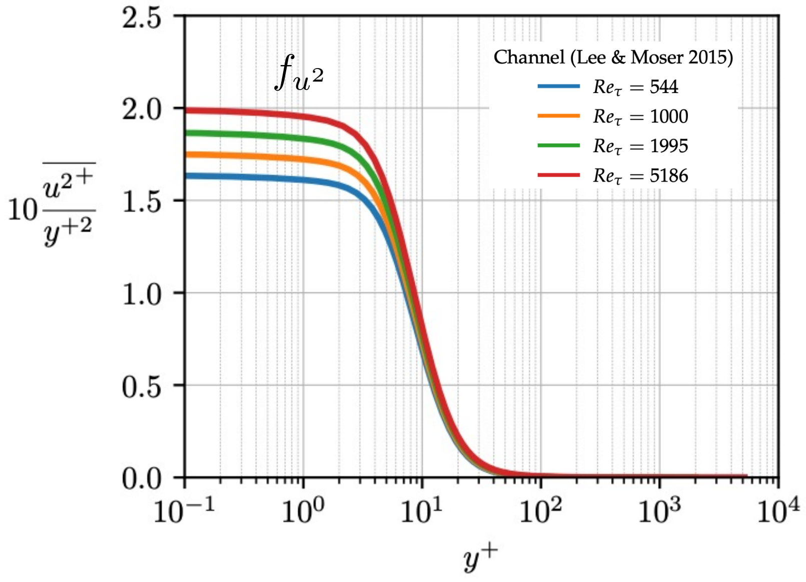
f_{u^2} $(= \overline{b_1^2})$ ➡ find using DNS

Smits, Hultmark, Lee, Pirozzoli & Wu (2021)

Channel flow DNS

$Re_\tau = 544,\ 1000,\ 1995,\ 5186$

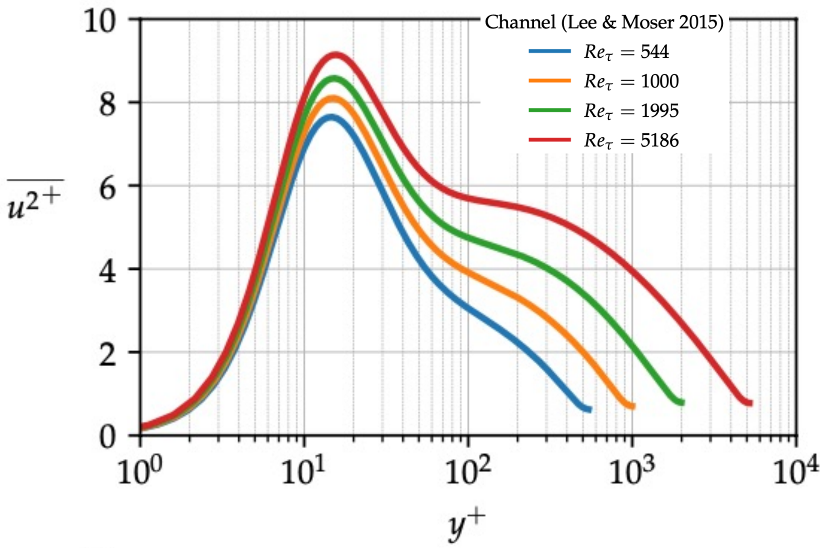


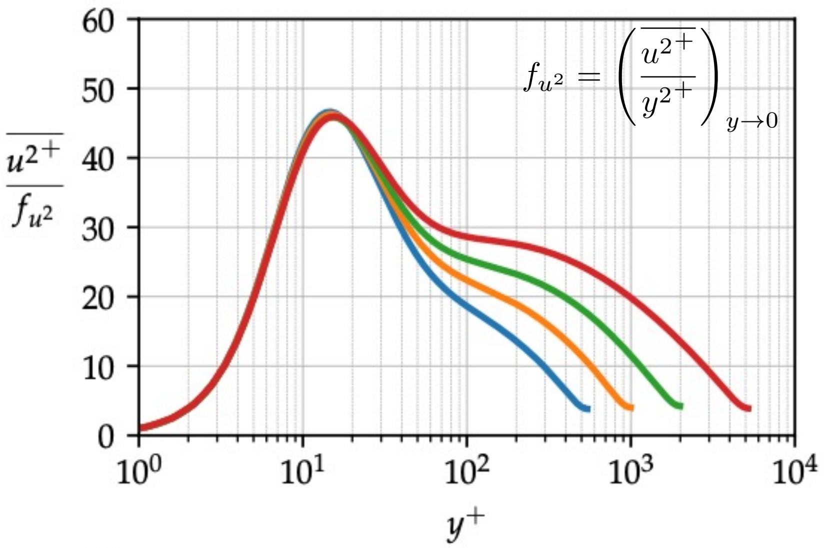


Data from Lee & Moser (2015)

Channel flow DNS

f-scaling





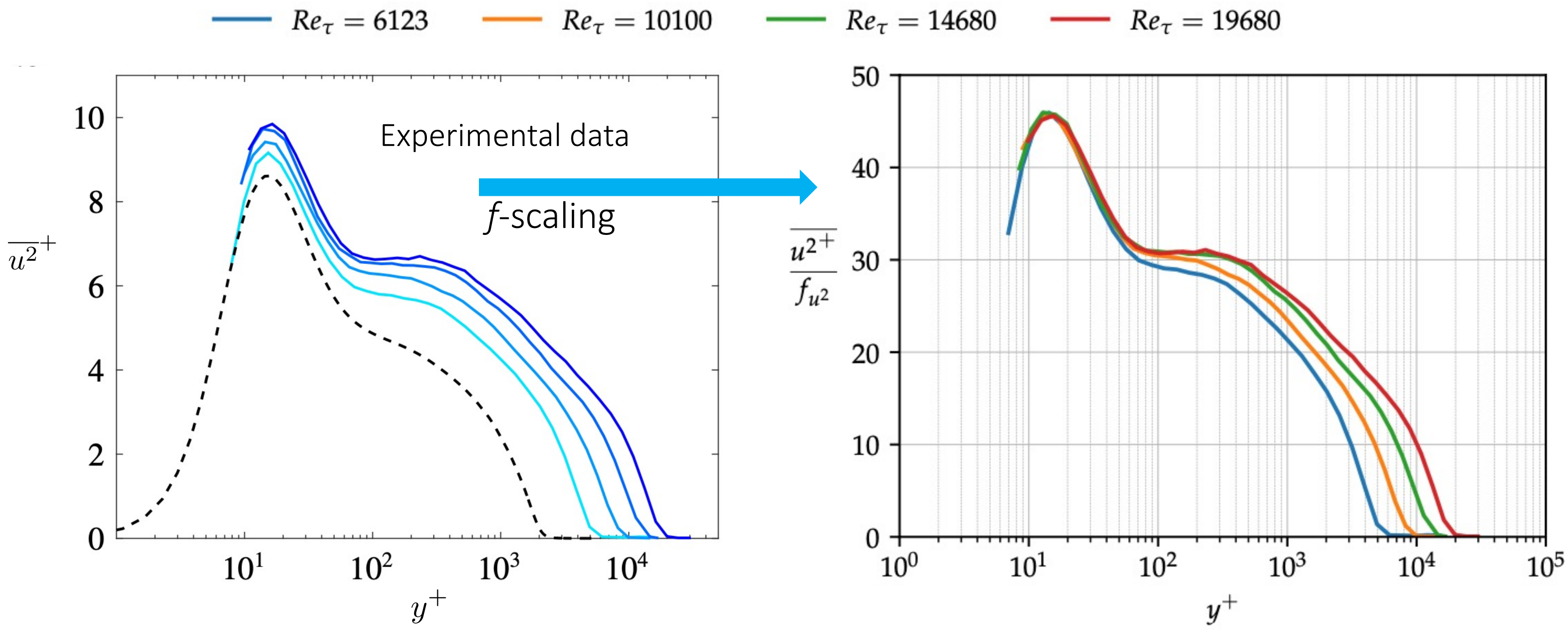
Data collapse for $0<y^+<20$, including peak

$$\overline{u^2}^+_p = 46 f_{u^2}$$

(see also Chen & Sreenivasan 2021)

Experiments at high Reynolds number

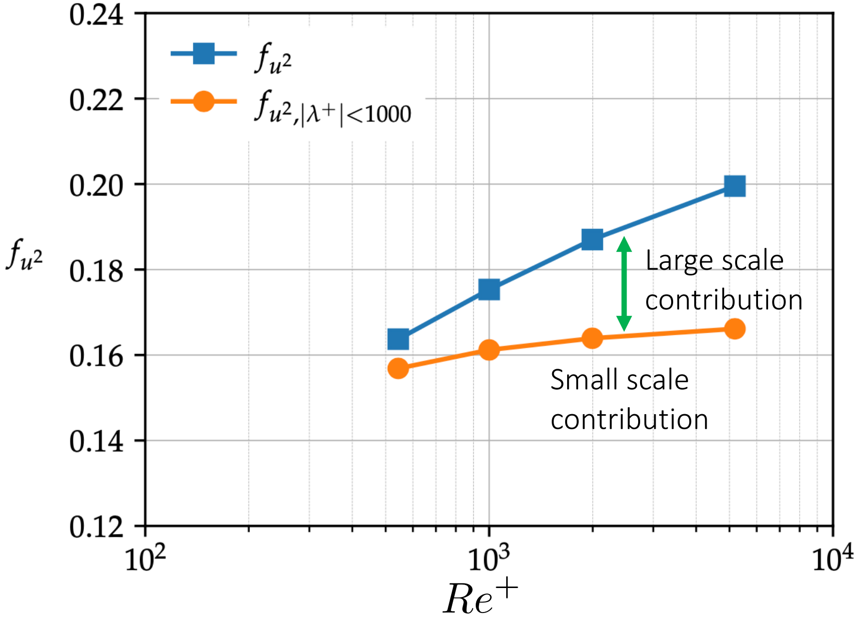
Melbourne boundary layer experiment (Samie et al. 2018)



Data collapse for $0<y^+<50$

What does it mean?

Scale decomposition



$$f_{u^2} = \overline{\left(\frac{\partial u^+}{\partial y^+}\right)_w^2} = \frac{\overline{\tau'^{\,2}_{wx}}}{\tau_w^2}$$

- Wall stress scaling
- Modulation and superimposition of the near-wall motions by large outer scale motions
- Determines scaling for entire near-wall region

Marusic et al. (2010)
Örlü & Schlatter (2011)
Mathis et al. (2013)
Agostini & Leschziner (2016, 2018)
Yang & Lozano-Duráan (2013)
Lee & Moser (2019)

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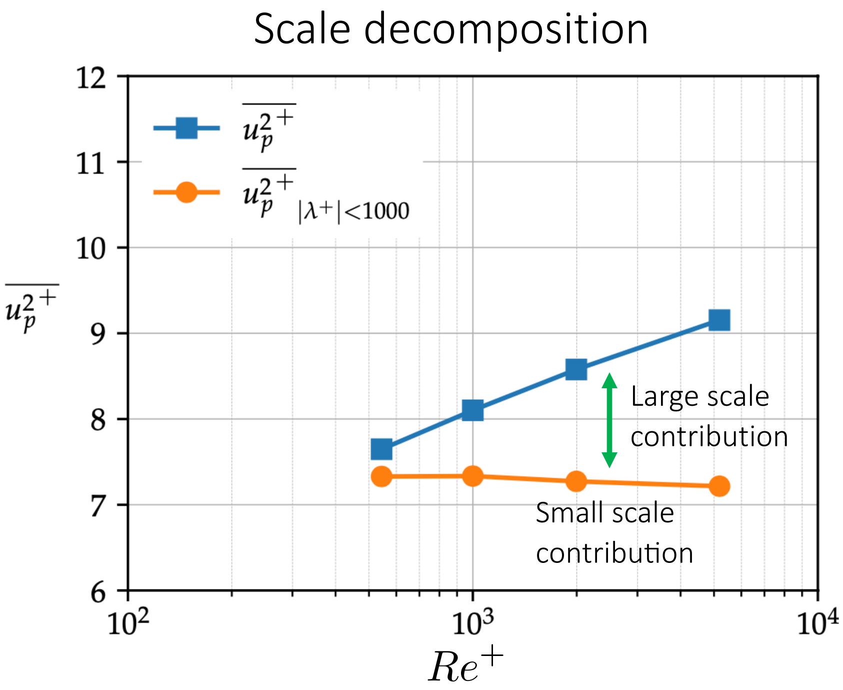

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Lee & Moser (2019)

Summary

- Pipe, channel and boundary layer flows obey similar scaling
- Some differences exist, primarily in the nature of the outer-layer eddy structure

Mean flow

- Inner and outer scaling
- Log-law widely accepted but only appears at high Reynolds number
- Power law blends viscous sublayer to log law
- Outer layer has two velocity scales at low Reynolds number, u_τ and u_{ZS}

Turbulence

- Outer scaling works well
- In overlap region, log-law in u^2 and w^2, but v^2 and –uv are constant
- Near wall intensity in u^2 grows with Reynolds number due to modulation and superimposition of the near-wall motions by large outer scale motions
- Wall stress determines scaling for entire near-wall region
- Outer peak in u^2 appears at high Reynolds number

Summary

Reynolds number scaling

- Need $Re_\tau > 10{,}000$ to understand high Reynolds number behavior
- Experiments were the only way to get high Reynolds numbers, but DNS is coming along (quickly)

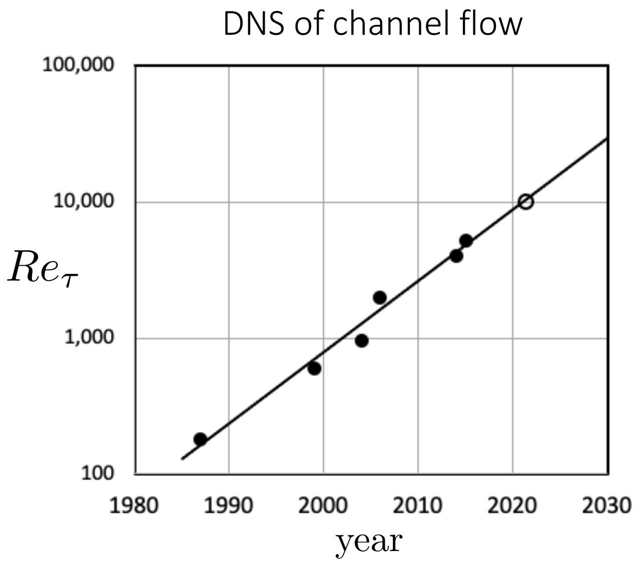
Summary

Reynolds number scaling

- Need $Re^+ > 10{,}000$ to understand high Reynolds number behavior
- Experiments were the only way to get high Reynolds numbers, but DNS is coming along (quickly)

Effort ~ $O(Re_\tau^3)$ for IHT

Coleman & Sandberg (2010)



Computing turbulent wall-bounded flows

Direct Numerical Simulations (DNS)

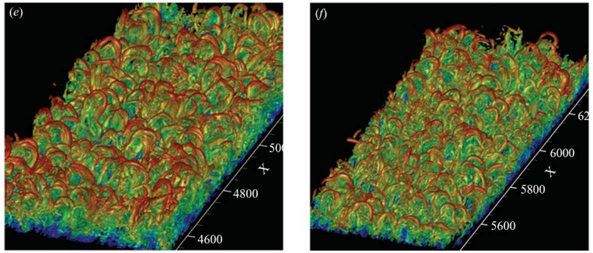
- 3D, time-resolved
- No model: captures all scales
- Accuracy limited only by numerical scheme, grid size/spacing, domain size
- Expensive, slow
- Cost > $O(Re^3)$
- Research tool

Large Eddy Simulations (LES)

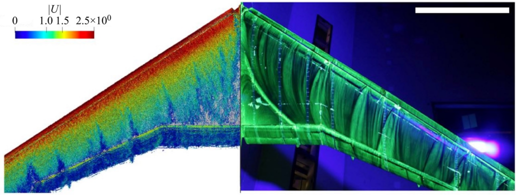
- 3D, time-resolved
- Model sub-grid scales
- Accuracy limited by model, grid size/spacing, domain size, wall treatment
- Medium expensive, medium fast
- Cost (wall-modeled) ~O(Re)
- Research tool transitioning to a design tool (Goc et al. 2021)

Reynolds-Averaged Navier-Stokes (RANS)

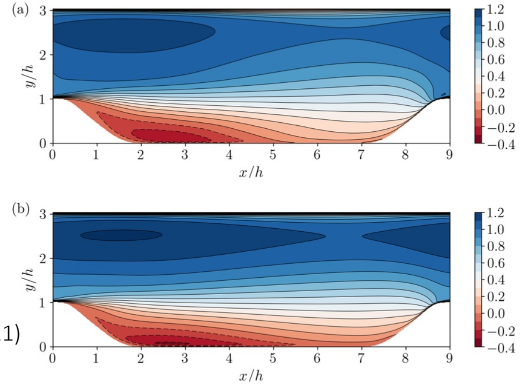
- 3D, steady or quasi-steady
- Model all turbulent scales (Boussinesq)
- Accuracy limited by model, numerical scheme, grid size/spacing, wall-functions, etc. (ERCOFTAC)
- Cheap, fast
- Design tool for industry



Wu & Moin (2009)



Goc et al. (2021)



DNS

RANS

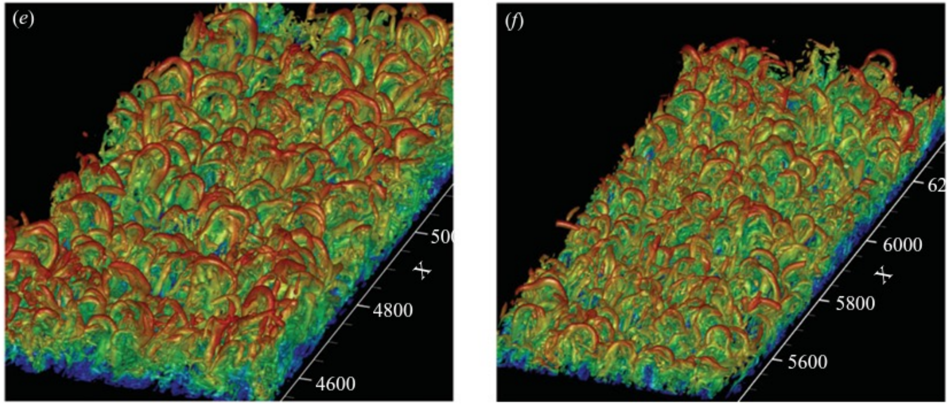
Volpiani et al. (2021)

Direct Numerical Simulations

- Very useful for examining near-wall behavior, although DNS Reynolds numbers) the outer layer influence is muted
- Captures scaling of spanwise and wall-normal stresses)
- In-depth spectral analysis (e.g., 2D spectra, true wavenumber spectra)
- Great for testing spatial and temporal content
- Not very useful for examining high Reynolds number behavior (e.g., log law constants, log-law in streamwise stress, inner peak, outer peak)
- Need Re_τ > 10,000 to understand high Reynolds number behavior

DNS of boundary layer flow

2.1 x 10^8 points, Re_τ max 460



Wu & Moin (2009)

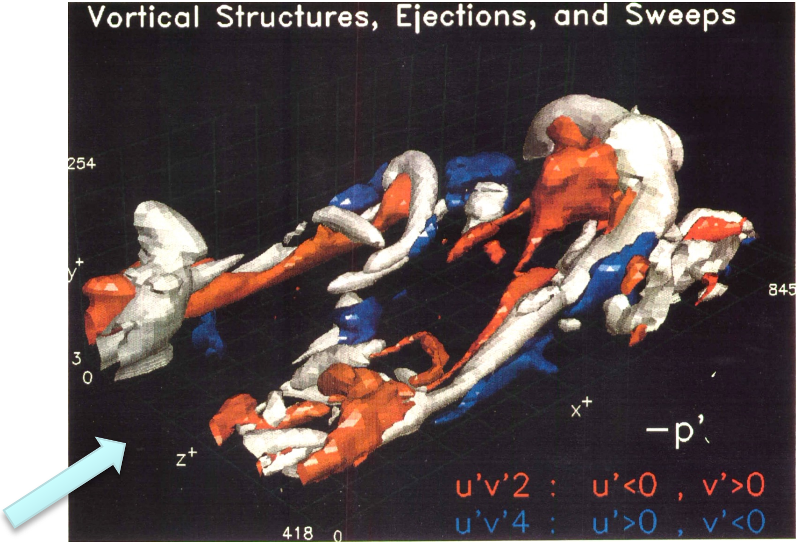
Effort ~ $O(Re_\tau^3)$ for IHT

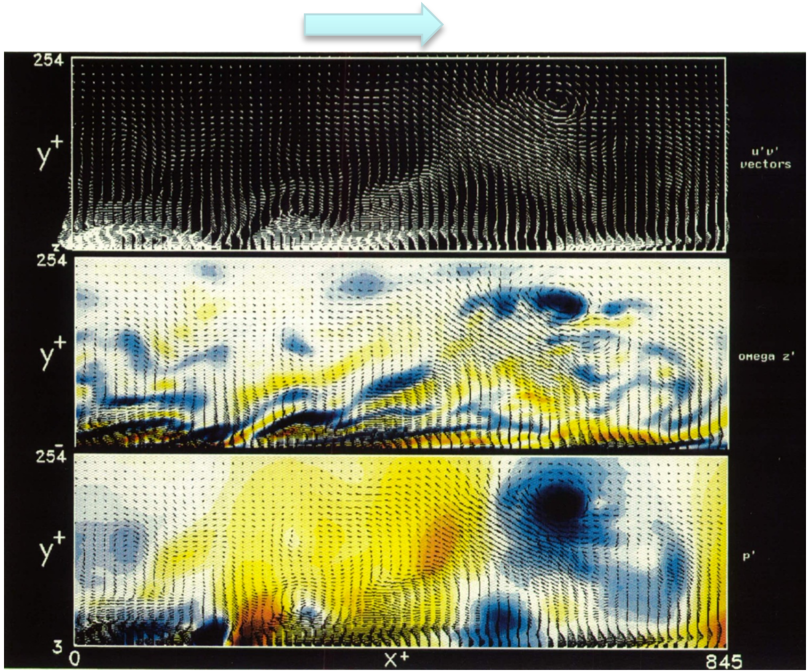
$Re_\tau = 170,\ 300,\ 650$ (Spalart 1988)

First DNS of a wall-boundary flow

Time evolution of channel flow with periodic boundary conditions

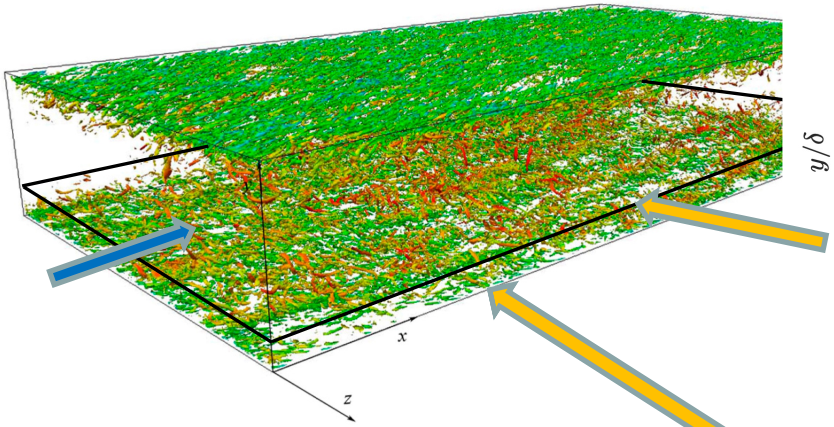
Robinson (1991) Re_τ = 300 case



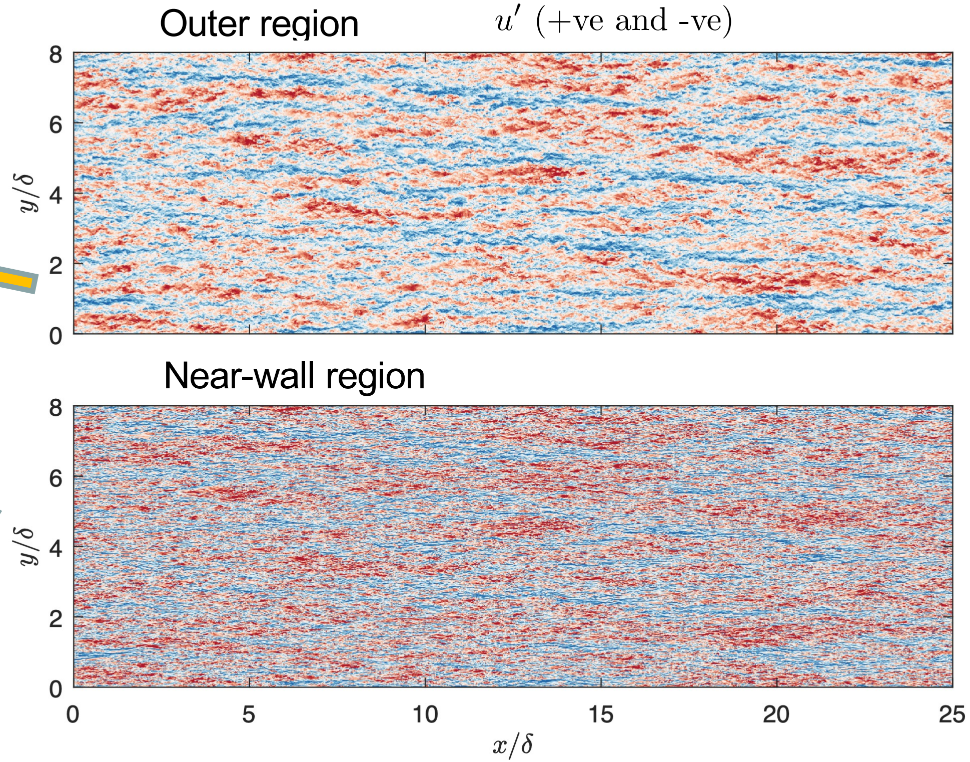


3.2 to 9.4 x 10^6 points

$Re_\tau = 2000$ (Hoyas & Jiménez 2011)



1.8 x 10^{10} points



Direct Numerical Simulations

Channel flow (Re_τ = 10,000)

Domain 8πh x 2h x 3πh

Pipe flow (Re_τ = 6000)

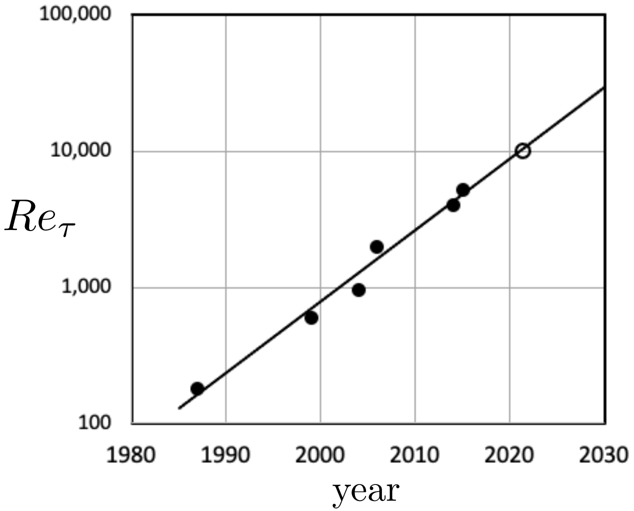
Domain 10-15πR

Boundary layer (Re_τ = 2000)

Transition

Typical resolution near the wall $\Delta x^+ = 10$, $\Delta y^+ = 0.2$, $\Delta z^+ = 5$

DNS of channel flow



Need Re_τ > 10,000 to understand high Reynolds number behavior

Large Eddy Simulations

Example: channel flow

Domain 4πh x 2h x 2πh

Wall-resolved (WRLES):

Turbulence resolved all the way to the wall, with typical resolution near the wall $\Delta x^+ = 15$, $\Delta y^+ = ??$, $\Delta z^+ = 20$

Computational cost ??

Wall-modeled (WMLES):

Turbulence not resolved for $y/\delta < 0.1$-0.2

For $y/\delta < 0.1$-0.2, use a wall-stress model, or a RANS model

ABL solvers typically impose a log-law for the mean flow, since the first grid point is typically already in the log region

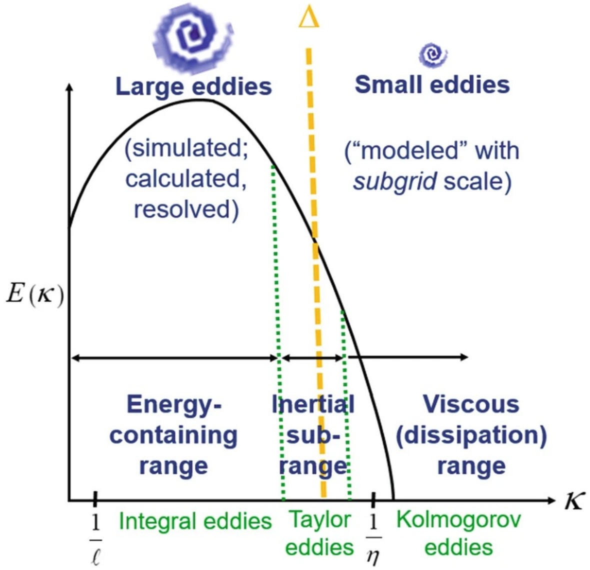
Computational cost ??

Need to choose a filter function with length scale Δ

Many choices: box, spectral, Gaussian, etc.

Need to choose a sub-grid scale model

Example, dynamic Smagorinsky



Rodriguez (2019)

Large Eddy Simulation milestones/people

Smagorinsky (1963) atmospheric flows

Deardorff (1970) channel flow

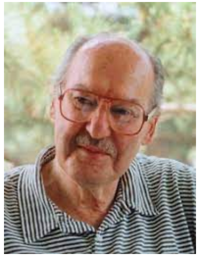
Schumann (1975)

Leonard (1975)

Kim & Moin (1979)

Piomelli (1989) – wall modeling

Spalart et al. (1997) – detached eddy simulation



Joseph Smagorinsky

James Deardorff

Ulrich Schumann

Anthony Leonard

John Kim

Parviz Moin

Ugo Piomelli

Philippe Spalart

Large Eddy Simulations

Consider the unsteady, incompressible momentum equation for turbulent flow

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j}$$

Decompose velocity in large-scale component + small-scale component: $u_i = \overline{u_i} + u_i'$

Need to define a filter function (can be different in all three directions). The filter width Δ is typically 2x the grid spacing. Filtered equation:

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + 2\nu \frac{\partial \bar{S}_{ij}}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}$$

Leonard (1975): $\tau_{ij} = L_{ij} + C_{ij} + R_{ij}$

$$\begin{cases} L_{ij} = \overline{\bar{u}_i \bar{u}_j} - \bar{u}_i \bar{u}_j & \text{Leonard stresses (interactions among large scales)} \\ C_{ij} = \overline{\bar{u}_i u_j'} + \overline{\bar{u}_j u_j'} & \text{Backscatter stresses (interactions between large and small scales)} \\ R_{ij} = \overline{u_i' u_j'} & \text{Reynolds stress-like term (interactions among sub-filter scales)} \end{cases}$$

τ_{ij} needs to be modeled

Piomelli (1987)

Smagorinsky sub-grid scale model

Basically, an eddy viscosity approach: $\tau_{ij} = -\nu_T \bar{S}_{ij}$ $\qquad \bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$

Small-scale
- Production $= -\overline{u_i' u_j'}\,\bar{S}_{ij} = 2\nu_T\,\bar{S}_{ij}\bar{S}_{ij}$
- Dissipation $= c_1 (\overline{u_i' u_i'})^{3/2}/\ell$

$\nu_T = \ell^2 (\overline{u_i' u_j'})^{1/2}$ (production = dissipation)

Here, ℓ is a length scale representative of a SGS eddy, so $\ell \propto \Delta$

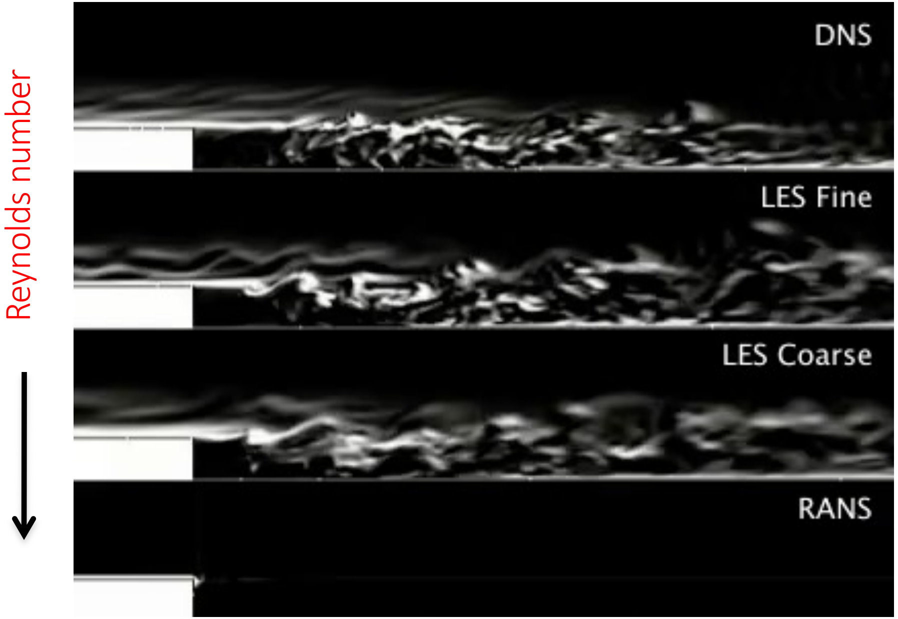
Hence, $\nu_T = C'\Delta^2 \sqrt{2\bar{S}_{ij}\bar{S}_{ij}} = C\Delta^2 |\bar{S}_{ij}|$ Smagorinsky-Lilly SGS model

Piomelli: $\ell_{PFM} = C_S \left[1 - \exp\left(-{y^+}^3/{A^+}^3\right)\right]^{1/2} (\Delta_1 \Delta_2 \Delta_3)^{1/3}$

which ensures the proper behavior for the SGS Reynolds stress τ_{12} near the wall ($\tau_{12} \sim {y^+}^3$)

Piomelli (1987)

Comparing turbulence models



DNS: “exact” solution

LES: solution without small eddies

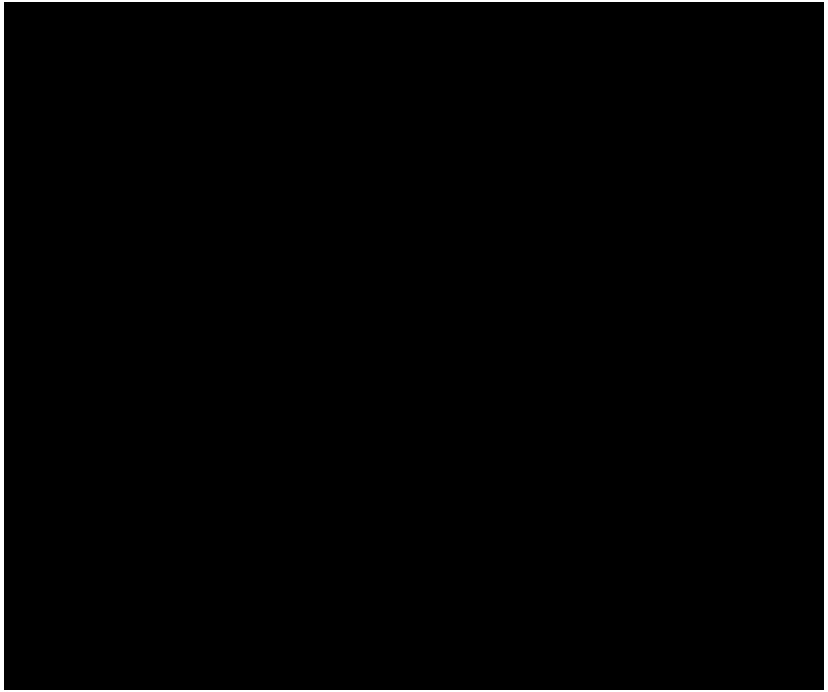
RANS: solution without any eddies (mean flow only)

At present, RANS is the best we can do for industrial flows

Rodriguez (2019)

Comparing turbulence models

Reynolds number



DDS

URANS: solution without small eddies

RANS: solution without any eddies (mean flow only)

At present, RANS is the best we can do for industrial flows

Reynolds-Averaged Navier-Stokes

- RANS models use Reynolds decomposition to derive equations for the mean momentum and the time-averaged turbulent stresses $\overline{u^2},\ \overline{v^2},\ \overline{w^2},\ -\overline{uv},\ \ldots$
- Essentially a steady flow model (although unsteady versions exist (URANS)
- Many different versions are in use
- All versions use the Boussinesq approximation where higher order quantities are modeled as gradients of lower order quantities (eddy viscosity models)
- All use the isotropic estimate for the dissipation

Two equation models

- One equation for turbulence
  - e.g., TKE (k-ε), vorticity (k, ω)
- One equation for length scale
  - Dissipation length scale L_ε

Reynolds stress models

- One equation for each turbulent stress component (up to 6 equations)
- One equation for length scale
  - Dissipation length scale L_ε

See also https://www.youtube.com/watch?v=zIQpxmLwbXQ&ab_channel=SteveBrunton

RANS: mean momentum equations

- For steady (or quasi-steady) flow, Reynolds decomposition gives the following mean momentum equations:

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}+W\frac{\partial U}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x}-\left(\frac{\partial \overline{u^2}}{\partial x}+\frac{\partial \overline{uv}}{\partial y}+\frac{\partial \overline{uw}}{\partial z}\right)+\nu\nabla^2 U$$

$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}+W\frac{\partial V}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y}-\left(\frac{\partial \overline{uv}}{\partial x}+\frac{\partial \overline{v^2}}{\partial y}+\frac{\partial \overline{vw}}{\partial z}\right)+\nu\nabla^2 V$$

$$U\frac{\partial W}{\partial x}+V\frac{\partial W}{\partial y}+W\frac{\partial W}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z}-\left(\frac{\partial \overline{uw}}{\partial x}+\frac{\partial \overline{vw}}{\partial y}+\frac{\partial \overline{w^2}}{\partial z}\right)+\nu\nabla^2 W$$

That is

$$U_j\frac{\partial U_i}{\partial x_j} = \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(-p\delta_{ij}+2\mu S_{ij}-\rho\overline{u_i u_j}\right)$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)$$

τ_{ij} needs to be modeled

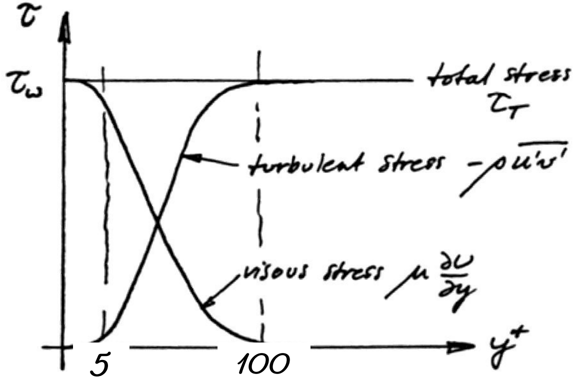
Boundary layer equations

- For steady flow, the 2D incompressible turbulent boundary layer equation is given by:

Total stress τ_T

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{dp_w}{dx} + \frac{\partial}{\partial y}\left(-\overline{uv} + \nu\frac{\partial U}{\partial y}\right)$$

Reynolds shear stress (needs to be modeled)



Near-wall region for zero pressure gradient flow

$$-\overline{uv} = \nu_t\frac{\partial U}{\partial y}$$ Prandtl's eddy viscosity

$$\frac{\tau_T}{\rho} = (\nu_t + \nu)\frac{\partial U}{\partial y}$$

In the log region $\frac{\nu_t}{\nu} \approx \kappa y^+$

RANS: turbulent stress equations

- For steady (or quasi-steady) flow, Reynolds decomposition gives the following turbulent stress equation:

$$\frac{D\left(\frac{1}{2}\overline{u^2}\right)}{Dt} = -\frac{1}{\rho}\frac{\partial \overline{p'u}}{\partial x} + \frac{1}{\rho}\overline{p'\frac{\partial u}{\partial x}} + \nu\overline{u\nabla^2 u} - \left(\overline{u^2}\frac{\partial U}{\partial x} + \overline{uv}\frac{\partial U}{\partial y} + \overline{uw}\frac{\partial U}{\partial z}\right) - \frac{1}{2}\left(\frac{\partial \overline{u^3}}{\partial x} + \frac{\partial \overline{u^2 v}}{\partial y} + \frac{\partial \overline{u^2 w}}{\partial z}\right)$$

Pressure diffusion (net loss of turbulent energy by work done in transporting fluid through regions of changing pressure gradient)

Tendency to isotropy (transfer of turbulent energy to other components)

Dissipation

Production (energy extracted from the mean flow by the turbulence)

Transport term (sums to zero across the shear layer)

Similar equations can be derived for the other stress components

Turbulence Kinetic Energy (TKE) equations

- Summing the normal stress equations gives the TKE or k-equation: $k = \frac{1}{2}\overline{q^2} = \frac{1}{2}\overline{u_i u_i} = \frac{1}{2}(\overline{u^2} + \overline{v^2} + \overline{w^2})$

Mean advection

$$\frac{D\left(\frac{1}{2}\overline{q^2}\right)}{Dt} = -\frac{\partial}{\partial x_j}\left(\frac{\overline{p' u_j}}{\rho} + \frac{1}{2}\overline{u_j q^2}\right) - \overline{u_i u_j}\frac{\partial U_i}{\partial x_j} + \nu\,\overline{u_i \frac{\partial^2 u_i}{\partial x_j^2}}$$

Pressure diffusion
(net loss of turbulent energy by work done in transporting fluid through regions of changing pressure gradient)

Turbulent advection
(rate of transport of TKE by the turbulence)

Production
(energy extracted from the mean flow by the turbulence)

Dissipation
(closely equal to the dissipation of mean flow kinetic energy into heat)

All terms on the RHS need to be modeled in terms of U_i and $\frac{1}{2}\overline{q^2}$

The Boussinesq approximation

- Boussinesq's hypothesis is that the turbulent stresses are related to the mean velocity gradients in a way that is similar to the way viscous stresses are related to the complete velocity gradients.

Incompressible form only

$$U_j \frac{\partial U_i}{\partial x_j} = \frac{1}{\rho} \frac{\partial}{\partial x_j} \left(-p\delta_{ij} + 2\mu S_{ij} - \rho \overline{u_i u_j}\right)$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

Definition of eddy viscosity

$$\overline{u_i u_j} = 2\nu_t S_{ij} - \tfrac{2}{3} k \delta_{ij}$$

$$k = \tfrac{1}{2}\overline{u_i u_i} = \tfrac{1}{2}(\overline{u^2} + \overline{v^2} + \overline{w^2})$$

2D example:

$$-\overline{uv} = \nu_t \frac{\partial U}{\partial y}$$

Prandtl's eddy viscosity

Definition of length scale

$$\nu_t \equiv \sqrt{k}\ell$$

2D example:

$$-\overline{uv} = \ell_m^2 \frac{\partial U}{\partial y}\left|\frac{\partial U}{\partial y}\right| \quad (\text{assuming } \overline{uv} \propto \overline{u^2})$$

Prandtl's mixing length

The underlying assumption is that ν_t or ℓ behave more simply than $\overline{u_i u_j}$

Mixing length and the log law

- Boussinesq's hypothesis is that the turbulent stresses are related to the mean velocity gradients in a way that is similar to the way viscous stresses are related to the complete velocity gradients.

Definition of length scale

$$\nu_t \equiv \sqrt{k}\ell$$

2D example:

$$-\overline{uv} = \ell_m^2 \frac{\partial U}{\partial y}\left|\frac{\partial U}{\partial y}\right| \quad (\text{assuming } \overline{uv} \propto \overline{u^2})$$

Prandtl's mixing length

Constant stress region:

$$-\overline{uv} = u_\tau^2 = \ell_m^2 \frac{\partial U}{\partial y}\left|\frac{\partial U}{\partial y}\right|$$

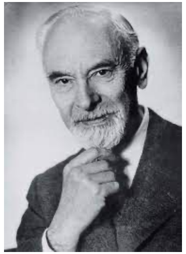
Hence:

$$\frac{\partial U}{\partial y} = \frac{u_\tau}{\ell_m}$$

Log law:

$$\frac{\partial U}{\partial y} = \frac{u_\tau}{\kappa y}$$

$$\ell_m = \kappa y$$



Ludwig Prandtl

Example: k-ε method

- Use the Boussinesq approximation to model the TKE equation in terms of q:

$$\frac{D\left(\frac{1}{2}\overline{q^2}\right)}{Dt} = -\frac{\partial}{\partial x_j}\left(\frac{\overline{p'u_j}}{\rho} + \frac{1}{2}\overline{u_j q^2}\right) - \overline{u_i u_j}\frac{\partial U_i}{\partial x_j} + \nu\,\overline{u_i \frac{\partial^2 u_i}{\partial x_j^2}}$$

Brian Launder

For high Reynolds numbers

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right) + \nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} - \varepsilon$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right) + C_1\nu_t\frac{\varepsilon}{k}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} - C_2\frac{\varepsilon^2}{k}$$

$$\nu_t = C_\mu k^2/\varepsilon \qquad \nu_t \equiv \sqrt{k}\ell$$

$$C_\mu = 0.09, \quad C_1 = 1.44, \quad C_2 = 1.92, \quad \sigma_k = 1.00, \quad \sigma_\varepsilon = 1.3$$

For low Reynolds numbers (near the wall) some modifications are necessary to account for viscous effects

Launder & Spalding (1974)

Example: k-ε method

For low Reynolds numbers (near the wall)

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_j}\left(\left(\frac{\nu_t}{\sigma_k} + \nu\right)\frac{\partial k}{\partial x_j}\right) + \nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} - 2\nu\left(\frac{\partial k^{1/2}}{\partial x_j}\right) - \varepsilon$$

Introduced for computational reasons

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_j}\left(\left(\frac{\nu_t}{\sigma_\varepsilon} + \nu\right)\frac{\partial \varepsilon}{\partial x_j}\right) + C_1\nu_t\frac{\varepsilon}{k}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} - C_2\frac{\varepsilon^2}{k} - 2.0\nu\nu_t\left(\frac{\partial^2 U_i}{\partial x_i \partial x_l}\right)^2$$

Introduced to make constrain k near the wall

$$C_\mu = C_{\mu\infty}\exp\left[-2.5/\left(1 + R_t/50\right)\right], \quad C_1 = 1.44, \quad C_2 = C_{2\infty}\left[1.0 - 0.3\exp(-R_t^2)\right], \quad \sigma_k = 1.00, \quad \sigma_\varepsilon = 1.3$$

$$(R_t = k^2/\nu\varepsilon \sim \nu_t/\nu)$$

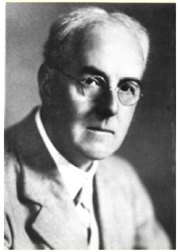
Launder & Spalding (1974)

Dissipation modeling

- RANS models the dissipation using the isotropic results as a basis:

RANS:

$$\left.\begin{aligned} \nu_t &= C_\mu k^2/\varepsilon \\ \nu_t &\equiv \sqrt{k}\ell \end{aligned}\right\} \quad \varepsilon = C_\mu \frac{k^{3/2}}{\ell}$$



Lewis Fry Richardson

- The Richardson energy cascade: *“Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity.”*

For high Reynolds numbers (large scale separation): $$\varepsilon = \frac{q^3}{\Lambda}$$

where q and Λ are the velocity and length scales characteristic of the energy containing motions

- Need the energy containing motions to be independent of the boundary conditions
- The flow must be in a state where the inertial region is fully established
- The ε-equation is actually a length scale equation

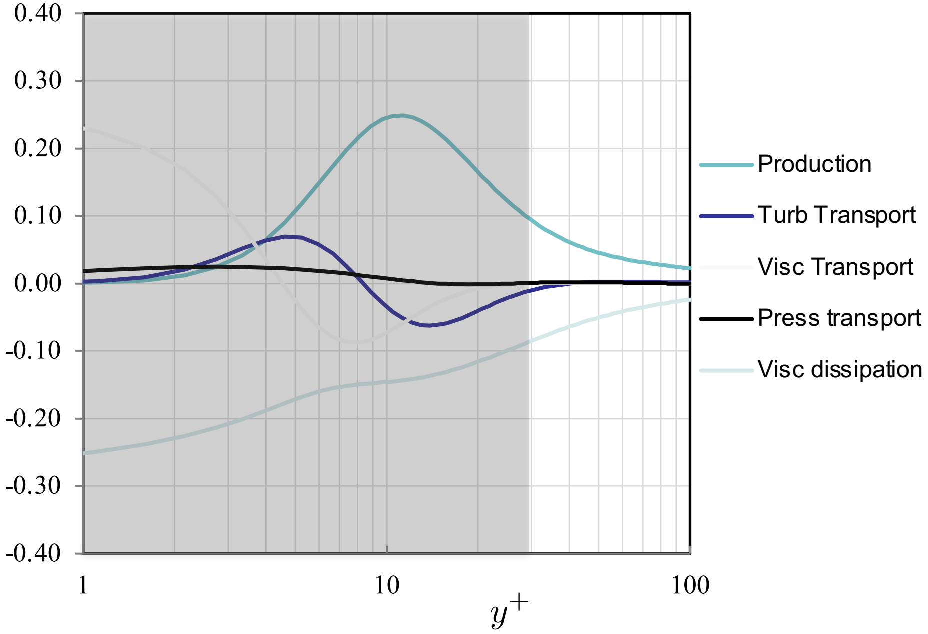
Andrey Kolmogorov

George Batchelor

Energy budget for k, Re_τ = 5200

$$0 = -2\overline{uv}\frac{dU}{dy} - \frac{d\overline{kv}}{dy} + \nu\frac{d^2k}{dy^2} + 0 + \frac{2}{\rho}\frac{d\overline{p'v}}{dy} - \epsilon_k$$

0 = production + turbulent transport + viscous transport + pressure strain + pressure transport + dissipation



Data from Lee & Moser (2015)

Energy budget for k, $Re_\tau = 5200$

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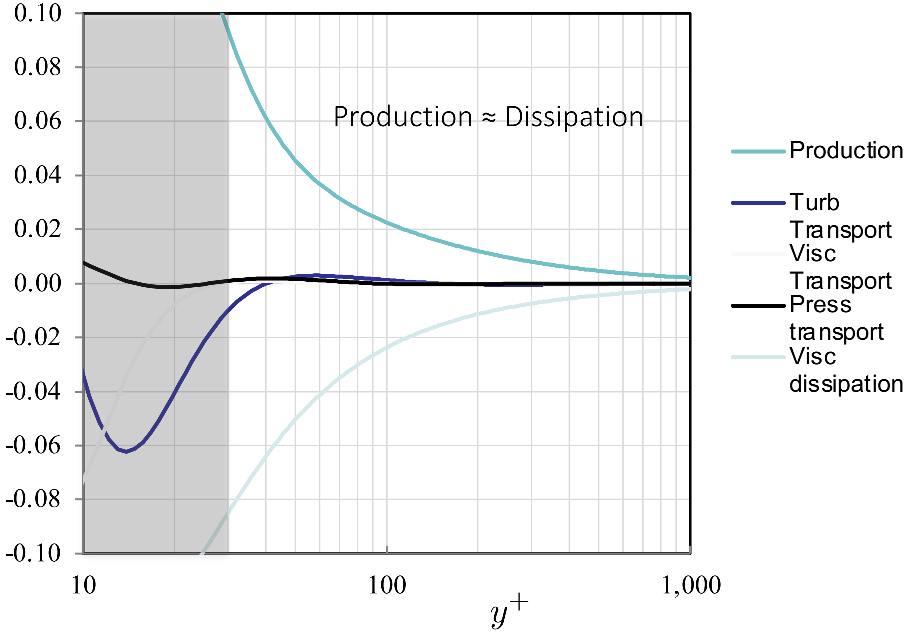
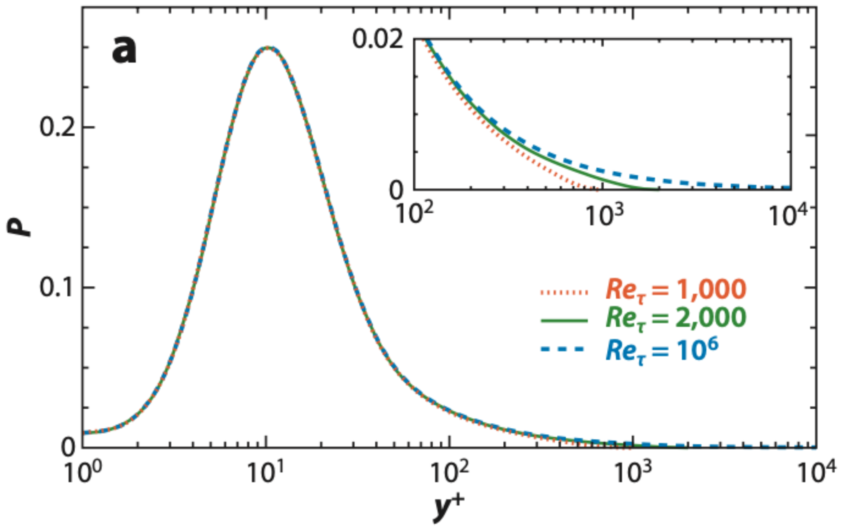


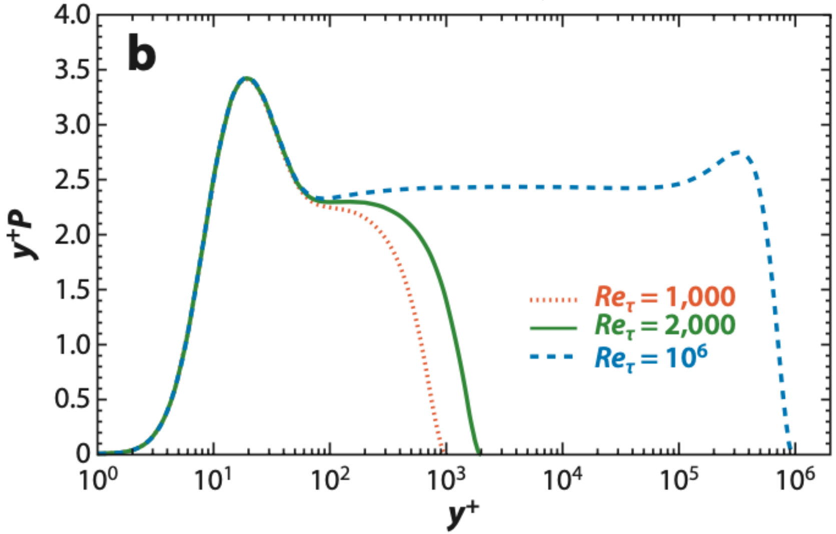
Data from Lee & Moser (2015)

Turbulence kinetic energy production

Semi-log



Pre-multiplied



$$P = -\overline{uv}\frac{dU^+}{dy^+}$$

Smits et al. (2018)

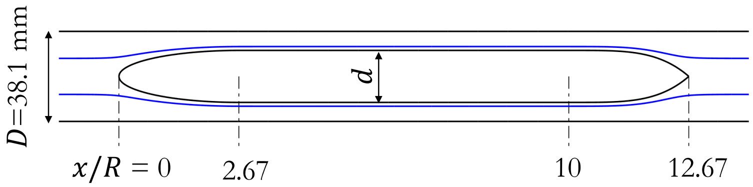
Summary: RANS methods

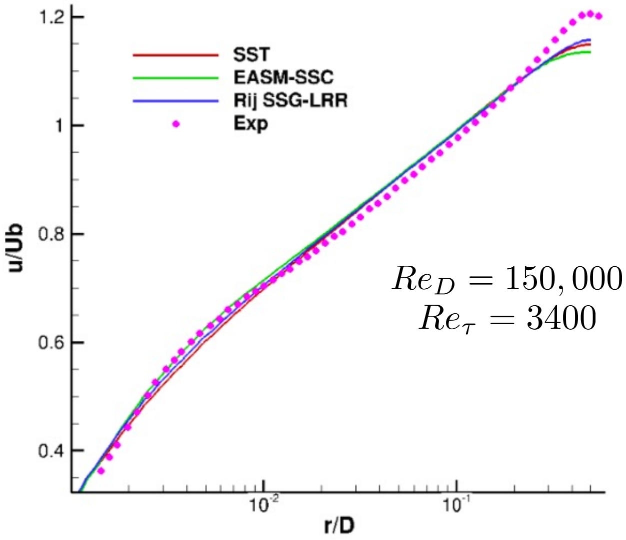
- Fast, stable, widely available commercially
- Industry standard design tool
- Ansys Fluent, OpenFOAM, SimFlow, Autodesk, FUN3D
- Many varieties available, tuned to specific flows (e.g., airfoils)
    - Spalart-Almares: transport equation for eddy viscosity (one-equation model tuned to airfoil flows)
    - k-ω: transport equations for k and ω $(\propto \varepsilon/k)$-- better near the wall than $k - \varepsilon$
    - Menter Shear Stress Transport (SST): switches from k–ω near the wall to k-ε away from the wall to get the best of both worlds
    - SSG-LRR: full Reynolds stress model using the Launder-Reece-Rodi pressure-strain model near the wall and the Speziale-Sarkar-Gatski model away from the wall
    - Etc.
- No method is very good at predicting separation on smoothly varying surfaces
- If there is defined separation point, then DES methods preferred
- ERCOFTAC, NASA, CFD Online, OpenFOAM, etc.
- Machine learning

See also https://www.youtube.com/watch?v=AgvjPPzy64I&ab_channel=SteveBrunton

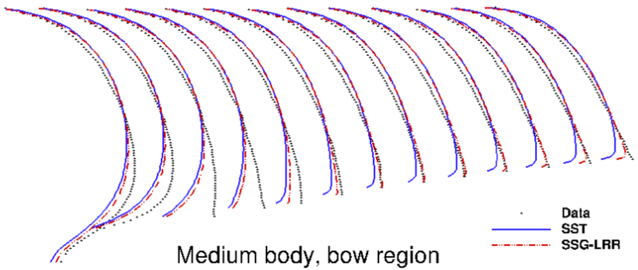
Example: RANS methods

- RANS has been used on almost every flow imaginable
- Sometimes it works, and sometimes it doesn't





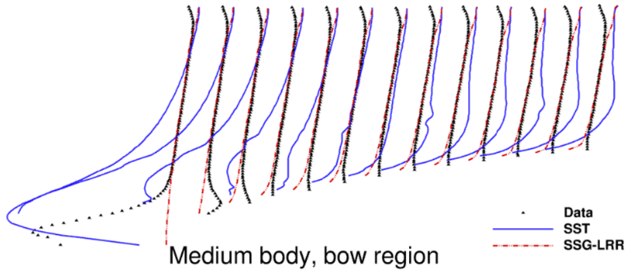
Mean velocity in empty pipe



Bow region

Mean velocity

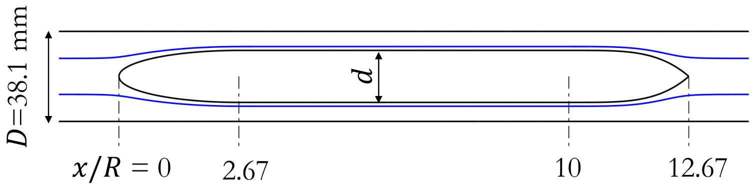
$-0.1 \leq x/R \leq 1.1$

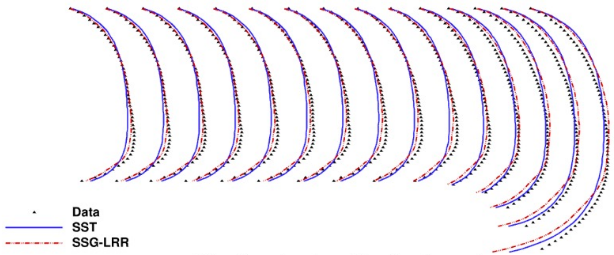


Shear stress

Vissoneau, Deng, Gilmineau, Ding, Smits (2022)

Example: RANS methods



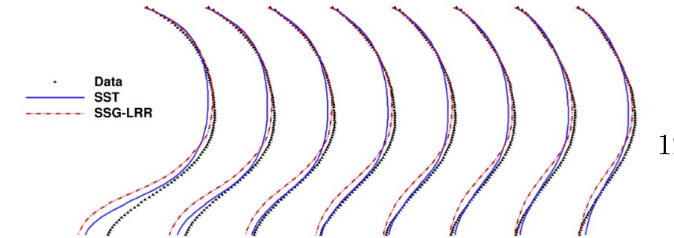


Medium body, after body region

Stern region

Mean velocity

$8.7 \leq x/R \leq 12.2$

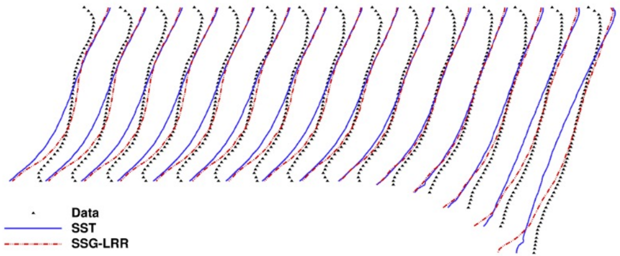


Medium body, wake region

Wake region

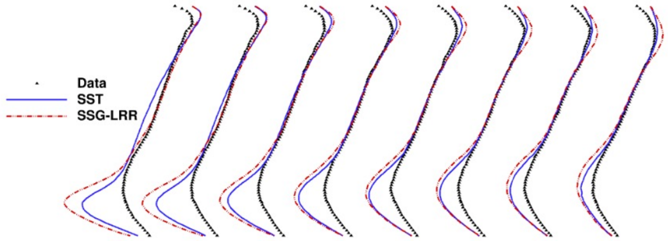
Mean velocity

$12.9 \leq x/R \leq 16.4$



Medium body, after body region

Shear stress



Medium body, wake region

Shear stress

Summary

- It is now obvious that fundamental studies of turbulence must be performed as a partnership between experiment and DNS
- Some questions can still only be answered by experiment
- Some questions can only be answered by DNS
- For canonical flows, DNS will very soon provide the necessary information for future understanding, instead of experiment

Summary

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- Some questions can only be answered by DNS
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- Beyond canonical flows:
    - Subsonic: pressure gradient, curvature, divergence, sudden perturbations …
    - High Mach number: heat transfer, chemistry, reacting flows

Summary

- It is now obvious that fundamental studies of turbulence must be performed as a partnership between experiment and DNS
- Some questions can still only be answered by experiment
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- Beyond canonical flows:
    - Subsonic: pressure gradient, curvature, divergence, sudden perturbations …
    - High Mach number: supersonic and hypersonic flows, shock-wave boundary layer interactions….

Questions?