# 4. Compressible flows

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# High speed flows



Smith 1989 (Mach 2.5)



Side view



Plan view





Space Shuttle (first flight 1981)



NASP X-30 (proposed 1986)



AF X-37B (first flight 2010)



Curiosity Mars probe (2012)



NASA X-43A/Hyper-X (first flight 2004)



DARPA Glide Breaker



Boeing X-51A Waverider (first flight 2010)

# Hypersonic vehicle flow fields



(Aerojet Rocketdyne)

(Anderson, 1989)

# Mean flow in high-speed boundary layers



- Mean velocity profile looks similar to that in incompressible flow but scales differently
- Temperature increases near the wall
- Density decreases near the wall
- Need to account for density changes

# Skin friction

#### Zero pressure gradient



 $\rho_w$  decreases with Mach number



 $\rho_w$  increases with compression

#### Adverse pressure gradient

#### Compressible turbulent flows and Reynolds number

- In a compressible flow, we have a choice of Reynolds numbers
- Friction Reynolds number

$$Re_{\tau} = \frac{\delta}{\nu_w/u_{\tau}} = \frac{\delta u_{\tau}}{\nu_w}$$

• Momentum thickness Reynolds number (freestream viscosity or wall viscosity)

$$Re_{\theta} = \frac{U_e \theta}{\nu_e}$$
  $Re_{\theta_w} = \frac{U_e \theta}{\nu_w}$   $(Re_{\theta} > 200 Re_{\theta_w} \text{ for Mach 10 helium flow})$ 

$$Re_{\delta_2} = \frac{\text{highest momentum flux}}{\text{highest shear stress}} = \frac{\rho_e U_e \theta}{\mu_w}$$

 $(Re_{\theta} > 10 Re_{\delta_2} \text{ for Mach 10 helium flow})$ 

• The inner layer thickness relative to the boundary layer thickness increases with Ma

#### Compressible turbulent flows and time scales

- The friction Reynolds number is often interpreted as a ratio of length scales
- What about time scales?

• Outer time scale: 
$$\frac{\delta}{U_{\infty}}$$
 Inner time scale:  $\frac{\nu_w}{u_{\tau}^2}$   
• Ratio outer to inner:  $R_{\tau} = \frac{u_{\tau}}{U_{\infty}} \left(\frac{\delta u_{\tau}}{\nu_w}\right) = \sqrt{\frac{C_f}{2}} Re_{\tau}$ 

• Time scale ratio decreases with Mach number

# What do we know or expect for the mean flow?

1. van Driest transformation collapses the mean velocity profile onto the incompressible form of the log law, and the incompressible form of the wake

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- In the overlap between the inner and outer scaling, we have a log-law
- Best expressed as

$$\frac{\partial \overline{U}}{\partial y} = \frac{\sqrt{\tau_w/\rho}}{\kappa y} \quad \longleftarrow \quad \text{velocity scale} \quad \text{length scale}$$

• Hence:

$$U_{VD} = \int_{U_1}^U \sqrt{\frac{T_w}{T}} dU = \int_{U_1}^U d\left(\frac{U}{u^*}\right)$$

where  $u^* = u_{ au} \sqrt{(
ho_w/
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(See also Lee, Helm, Martin, Williams 2023)



Williams, Sahoo, Baumgartner & Smits (2018)

# What do we know or expect for the turbulence?

2. Mach number effects act only through the mean density (temperature) variation, and effects due to density fluctuations (shocklets, for example) are negligible even at high Mach number

Mie scattering from acetone droplets





Smith (1989); Smith & Smits (1995)

Smoke flow visualization



### What do we know or expect for the turbulence?

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Rayleigh scattering (FRS):

- Scattering from particles with d <  $\lambda$
- ~10 nm clusters of H<sub>2</sub>O, condensed in nozzle from naturally occurring water in air supply, or
- ~10 nm clusters of CO<sub>2</sub>, condensed in nozzle from CO<sub>2</sub> gas injected upstream (Miles & Forkey 1991)



Mach 2.9,  $Re_{\theta} = 78,000$ 



Side view

Plan

view

# Scaling the turbulence

3. Morkovin scaling collapses the turbulent stress profiles onto the incompressible data, at least for the overlap region and the outer flow

Mach = 8 Mach = 0



FRS – Baumgartner et al. (1997); PLIF - Delo & Smits 1997)

# Scaling the turbulence

3. Morkovin scaling collapses the turbulent stress profiles onto the incompressible data, at least for the overlap region and the outer flow

Morkovin's hypothesis (1961):

• "The essential dynamics of these shear flows will follow the incompressible pattern"

$$u_{\tau} = \sqrt{\tau_w/\rho_w}$$
$$\downarrow$$
$$u^* = \sqrt{\tau_w/\rho}$$

Mach = 8 Mach= 0



FRS – Baumgartner et al. (1997); PLIF - Delo & Smits 1997)

## Scaling the turbulence

3. Morkovin scaling collapses the turbulent stress profiles onto the incompressible data, at least for the overlap region and the outer flow



# Historical record

Including all experimental data, up to Ma = 11, obscures any conclusion

Previous hot-wire measurements often suffer from

- inadequate frequency response
- mixed-mode response/calibration
- inadequate spatial resolution.



Williams, Sahoo, Baumgartner & Smits (2018)

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What about PIV at high Mach number?



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# obviously the correct signal on which to base measure-Practically, it was impossible to perform two-dimen-

ments. In both methods, the entire image is divided into a grid of (usually overlapping) interrogation spots, and the particle images in each spot are interrogated to obtain the mean displacement of the particles within each interrogation cell, willch consists of the intersection of the interrogation spot area,  $A_{\rm I}$ , and the thickness of the light sheets Azar. Analysis of the auto-correlation method (Keane and Adrian 1992) led to the definition of a second dimensionless number, called the *image density*. It is equal to the average number of scattersheets interrogation cell. This number proved to be very important in describing the characteristics PIV systems and in optimizing their design. The low image density limit corresponds to particle tracking, becaused luminated mit, it is improbation of the spot of The High track patters to function of the multiple • First light pulse patticle correlation PIV (Fig. 4). • Second light pulse At the first daade of PIV, the greatest challenge was the interrogation of the images, simply because computer capabilities were not accuate for the task. In 1985, the  $_{\odot} DEC PDP^{x_{11}/23}$  was a common digital computer in

many fluids haboratories. It typically had 128 KB of RAM and a 30 MB hard drive. Imagine holding the operating system, the executable program, and the data in a RAM space that is the same size as the minimum document file size used by current word processors.



sional Fourier transforms or two-dimensional correlation analysis on such machines. Therefore, there was considerable interest in non-statistical methods, such as tracking particles individually. Alternatively, several groups. sqringer purtue of the spectrum as a religion of a wast. twodimensional correlations by analog optical means (Morck et magge Game et al. 1996). Particle tracking implied operations the two most operations of the transition of tr probability of finding more than one pair of particles per interrogation spot was small. Then, using the principle that nearest-neighbor images corresponded to the same particle (which is only approximate for small, but finite image density), one could make successful measure ments. The difficulty with the smethod was that at the reduced image density that accompanied reduced particle concentration, the number of escapation into a feation not large enough to repay the subrefields for house the flow To improve the spatial resolution, various investiga to optimize the low image density method by using interrogation windows of variable size, shape, and displacement. This led to the implementation of adaptive windowing methods. Currently, adjustable window methods enjoy use as a means of optimizing singleexposed double-frame images obtained with digital cameras.

At the time that Meynart performed his work using Young's fringes, the dynamic velocity range of the technique, defined as the maximum velocity measurable divided by the minimum velocity measurable, was somewhere between 5 and 10 PIV was a velocity-mea-











- Results agree well with DNS at the same Reynolds number
- Show the expected behavior with increasing Reynolds number
- Support Morkovin scaling
- But...





- It is very difficult to measure the wall-normal component v'
- This is a general result for all experimental methods and almost all experiments



# What about DNS?



- DNS of turbulent boundary layers
- (small box DNS, ~10 $\delta$ , bigger box results soon)
- Matched Reynolds numbers ( $570 < Re^+ < 377$ )
- 0.3 < Ma < 11.9
- $1 < T_{w}/T_{\infty} < 28$

Profiles collapse in Morkovin scaling, for all three components, even at Ma = 12

#### Strong Reynolds Analogy between temperature and velocity



Smith & Smits (1993)

# Unform momentum zones (UMZ)



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# Superstructures (VLSMs)



### Beyond flat plate boundary layers

- Van Driest and Morkovin work well for flat plate boundary layers
- What about "real" world effects?
  - Tripping effects
  - Transitional effects
  - Surface curvature
  - Görtler vortices
- What about complex flows?
  - Shock wave boundary layer interactions
  - Superstructures
  - Görtler vortices

# Tripping effects



Subsonic flow Marusic et al. (2015)

P40 grit sandpaper, 6mm and 10 mm threaded rod, matched Re\_{\tau}  $\approx$  11000



# Tripping effects



• Hyper-X tripping at Mach 6,  $\alpha$  = 2°



Berry et al. (2001)



### Upstream history effects

- Periodic roughness upstream of the throat at Mach 2.3
- Vortex generation in subsonic flow
- Subsonic experiment, following development of streamwise vortices on a wind tunnel wall
- TG vortices form in the contraction, with  $\lambda \approx 2\delta$ , persisting into the working section through transition
- As the boundary layer grows, the vortices appear to pair so as to maintain  $\lambda \approx 2\delta$ , so their number decreases with downstream distance





Dussauge & Piponniau (2008)

# Transitional structures in boundary layers



- Organized, longitudinal structures in turbulent boundary layer appear to originate at specific spanwise locations
- Long-lasting

• Organized, longitudinal structures ripple the separation line ahead of cylinder interaction



Murphree et al. (2006)

# Shock-wave boundary layer interactions (SWBLI)

- Pressure gradient
- Shock-induced separation
- Streamline curvature effects
- Transverse curvature effects
- Upstream history effects, initial conditions
- Reynolds number effects
- Unsteadiness
- Three-dimensionality
- Asymmetry

Dolling 2001; Délery & Dussauge 2009; Babinsky & Harvey 2011; Gaitonde 2015



Dupont et al. (2005) 9.5° reflected shock at Mach 2.3

# Shock-wave boundary layer interactions (SWBLI)

- Intense shock unsteadiness
- Shock splits and ripples
- Strong shocks lead to separation
- Shock interacts with boundary layer to amplify turbulence
- Peaks occur in friction and heat transfer



### Unsteadiness of SWBLI and superstructures

- Shock motion in SWBLI increases with shock strength
- Can lead to severe unsteady pressure and heat loading
- In unseparated flows, VLSM + TG vortices
- In separated flows, VLSM + shear layer instability + TG vortices



Fig. 1 Typical separated compression ramp flowfield surface properties: a) pressure signal near separation; b) intermittency; and c) mean wall pressure and standard deviation distributions.<sup>3</sup>





plan view

SWBLI at Mach 2.5

500 KHz sequential images of using FRS (Miles et al.)

MHz rate pulse-burst laser, coupled with MHz rate CCD framing camera (PSI)





#### FRS visualizations $8^{\circ}$ compression corner at Ma = 7.2

Filtered

Rayleigh



Bookey, Wyckham, Smits & Martin, AIAA 2005

# PIV data for $8^{\circ}$ and $33^{\circ}$ compression corners at Ma = 7.2













- Progressively larger amplification through the shock
- Morkovin scaling no longer works





- Progressively larger amplification through the shock
- Morkovin scaling no longer works



#### Superstructures in supersonic flows



#### Superstructures and SWBLI





Superstructures modulate the shock and cause unsteadiness (unseparated flows)

Bookey et al. (2005); Schreyer et al. (2011); Priebe & Martin (2021)

# Unsteadiness of separated SWBLI

- Shock motion and unsteadiness in SWBLI increases with shock strength
- In unseparated flows, the superstructures appear to be the prime cause of the shock motion (relatively high frequencies)
- In separated flows, there is a large-scale, slow pulsation of the separation bubble (relatively low frequencies)
- Pulsation related to the entrainment of fluid into the mixing layer feeding the separation bubble
- Leads to a mass balance model of SWBLI interactions and a separation criterion based on Ma, Re, and turning angle



#### Concave streamline curvature and Görtler vortices



Smits, Young & Bradshaw (1979)

200 250 300 1350

#### Concave streamline curvature and Görtler vortices



Mach 3.2



Roshko & Thomke (1966) Shamroth & MacDonald (1970)



SWBLI Mach 2.9, 8° to 24° Settles et al. (1979)





"Therefore, although one can conceive that orderly arrays of pairs of counter-rotating vortices of the Gortler type, existing in the free shear layers as a result of streamline curvature, imply this type of flow attachment, one can also conceive that vortices are being developed as a result of this being the only possible type of attachment in the presence of any weak initial disturbance in the approaching boundary layer." (Ginoux 1971)

#### Superstructures and separated SWBLI



Turbulent boundary layer compression ramp interactions (LES)

Helm & Martin (2021)

# Schulein & Tromifov 2011 (Mach 5)

- Longitudinal vortices, initiated at the leading edge, survive for long distances, O(10<sup>4</sup>h)
- The total number, dimension and type of installed vortex generators define the number of vortex pairs generated downstream





## VG control for $33^{\circ}$ compression corner at Ma = 7.2



# Separation by reflected shock Mach 5.8



# Helium injection in Mach 8 turbulent boundary layer

![](_page_53_Figure_1.jpeg)

$$J = \frac{(\rho U^2)_{inj}}{(\rho U^2)_{\infty}} = \frac{(\gamma p M^2)_{inj}}{(\gamma p M^2)_{\infty}}$$

- Organized, longitudinal structures in turbulent boundary layer are formed in region of helium injection
- Taylor-Görtler vortices due to concave curvature caused by injection

![](_page_53_Figure_5.jpeg)

# Görtler number

• Taylor-Görtler vortices in <u>laminar</u>, incompressible flows with concave streamline curvature are seen to occur when (Liepmann, Schlichting, Dryden)

$$G_{\theta} = Re_{\theta} \sqrt{\frac{\theta}{R}} > [6,9]$$

• For <u>turbulent</u>, incompressible flows with concave streamline curvature, Tani (1962) suggested

$$G_{\theta} = \frac{U_{\infty}\theta}{\nu_T} \sqrt{\frac{\theta}{R}} = \frac{55}{H} \sqrt{\frac{\theta}{R}} > [\approx 4]$$
  
with  $\nu_T = 0.018 U_{\infty} \delta^*$  (Clauser)

- For turbulent, <u>compressible</u> flows with concave streamline curvature, Smits & Dussauge (2006) suggested incorporating compressible forms of H and θ
- For a given  $\delta,$  H increases faster than  $\theta$  decreases with M, so the Görtler number decreases with M
- Boundary layers more stable to concave curvature still holds with  $\nu_T \propto U_\infty \delta_i^*$  (Maise & McDonald)

![](_page_54_Figure_8.jpeg)

You et al. (2021)

Are these simple ideas correct?

# Separation on cylinder/flare Mach 5

0

0.5

M = 5, axisymmetric flare/cone Oil flow visualization in natural transition

![](_page_55_Picture_2.jpeg)

a)  $p_{st} = 0.9 \times 10^5$  Pa,  $Re_L = 0.38 \times 10^6$ 

![](_page_55_Picture_4.jpeg)

b)  $p_{st} = 2.1 \times 10^5$  Pa,  $Re_L = 0.67 \times 10^6$ 

![](_page_55_Picture_6.jpeg)

c)  $p_{st} = 5.4 \times 10^5$  Pa,  $Re_L = 1.46 \times 10^6$ 

![](_page_55_Picture_8.jpeg)

d)  $p_{\rm st} = 5.5 \times 10^5$  Pa,  $Re_L = 1.55 \times 10^6$ 

• Taylor-Görtler vortices present in all cases

$$G_{\theta} = Re_{\theta} \sqrt{\frac{\theta}{R}}$$

$$G_{\delta^*} = Re_{\delta^*} \sqrt{\frac{\delta^*}{R}} = H^{3/2}G_{\theta}$$

$$H \approx \left(1 + 0.693(\gamma - 1)M_{\infty}^2\right)\overline{H} = 20.5 \quad \text{at } M_{\infty} = 5$$

$$G_{\delta^*} \approx 90G_{\theta}$$

$$G_{\delta^*} \approx 90G_{\theta}$$

$$Re_L = 0.38 \times 10^6,$$

$$= 0.68 \times 10^6$$

X/L 1.5

Benay et al. (2006)

Skin friction at Ma = 8

![](_page_56_Figure_2.jpeg)

Roy & Blottner (2006)

![](_page_57_Figure_1.jpeg)

![](_page_57_Figure_2.jpeg)

Roy & Blottner (2006)

# 34° compression corner at Ma = 9.2 (separated)

![](_page_58_Figure_1.jpeg)

Roy & Blottner (2006)

- Zero pressure gradient, smooth wall flows appear to follow Van Driest/Morkovin scaling for mean flow and turbulence
- Shock-wave boundary-layer interactions are inherently unsteady, with large fluctuations in wall pressure and heat transfer
- SWBLI can cause very large amplifications of turbulence
- RANS models not reliable for SWBLI, especially at high Mach number

- Increasing Mach number generally coupled with decreasing Reynolds number
  - perfect for DNS (or high-resolution LES)
- High Mach number experiments must be designed for DNS
- Experiment and DNS/LES must be done at the same Reynolds number
- Need to expand our interests to include
  - heat transfer, roughness, film cooling
  - ablation, wall catalysis, pyrolysis, real gas effects, chemistry
  - curvature, pressure gradient, three-dimensionality
- Diagnostics are crucial to everything (understanding, modeling, validation, etc.), especially for turbulence
  - New diagnostics for  $\tau_w$ ', q', species, reaction rate?

# Opportunities

- Flows with pressure gradients
  - What constitutes a canonical experiment?
  - Is there one or more characteristic parameters?
  - Prediction of separation on smoothly varying surfaces is still a major challenge
  - Data base is scattered, not systematic nor complete
- Turbulence Modelling
  - LES for separated flows well-established
  - RANS-DES easy for fixed separation point, hard for smoothly varying surfaces
  - LES wall functions necessary to make progress lots of work needed
  - RANS for compressible flows, especially SWBLI (including transonic flows with possible separation)
  - Hypersonic flows
- Reduced order modeling
  - Resolvent analysis (McKeon)
  - Linear models (Gayme)
- Machine learning/AI
  - Can only be effective if there is a large-enough database

# Incompressible turbulence

- Canonical turbulent flows
  - Grid turbulence
  - Jets and wakes
  - Fully-developed flows (channel, pipe, Couette, Ekman, Taylor-Couette)
  - Flat plate boundary layers, smooth or rough walls
- Perturbations in boundary conditions
  - Roughness, heat transfer
  - Step changes, impulsive changes
- Pressure gradients
- Complex flows
  - Streamline curvature (convex and concave)
  - Streamline convergence and divergence
  - Rotation (turbulent vortex flows)
  - Stratification (stable or unstable)
- Bodies of revolution (concurrent complex flows)
- Separation
  - Sudden (backward facing step, sharp edges)
  - Gradual (slowly varying flows)
- Flows with chemistry (mixing)
- Noise generation and transmission
- Flow control

![](_page_62_Figure_22.jpeg)

![](_page_62_Picture_23.jpeg)

### Compressible turbulence

- Canonical turbulent flows
  - Grid turbulence
  - Jets and wakes
  - Channel, pipe
  - Flat plate boundary layers, smooth or rough walls
- Perturbations in boundary conditions
  - Roughness, heat transfer
  - Step changes, impulsive changes
- Pressure gradients
- Complex flows
  - Streamline curvature (convex and concave)
  - Streamline convergence and divergence
  - Rotation (turbulent vortex flows)
  - Stratification (stable or unstable)
- Separation
  - Sudden (backward facing step, sharp edges)
  - Gradual (slowly varying flows)
- Shock-wave boundary layer interactions (2D and 3D)
- Flows with chemistry
- Noise generation and transmission

![](_page_63_Picture_21.jpeg)

![](_page_63_Picture_22.jpeg)

Mach 5.6 (Canning, NASA-Ames)

![](_page_63_Picture_24.jpeg)

# Some opportunities in turbulence

- Scalar transport
  - Contaminants, food or prey, pheromones
- Two-phase flows
  - Particle/bubble tracking (Lagrangian dynamics)
  - Cloud formation, plankton upwelling
  - Sediment, dust transport
- Atmospheric/geophysical flows
  - Stable (night time), unstable (convective, day time)
  - Rotating flows (Coriolis)
  - Heat, mass and momentum fluxes
  - Step changes (leads and polynyas)
- Stratified turbulence

![](_page_64_Picture_13.jpeg)

Experiment, DNS, LES

Golden age of turbulence

Rapidly improving experiment and computation

Democratization of experiment and computation

New facilities, new ideas

Need courage to move beyond canonical flows

# Questions?

![](_page_65_Picture_1.jpeg)